LONDON MATHEMATICAL SOCIETY
EST. 1865

NEWSLETTER

Issue: 486 - January 2020

CELEBRATING DE MORGAN HOUSE

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EDITORIAL OFFICE

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Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

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FROM THE EDITOR-IN-CHIEF

Welcome to the January 2020 issue of the LMS *Newsletter*. Now I know that change is never easy, but here we have taken the brave step of beginning this issue by directly addressing you, the Reader. This isn't purely an innovation by myself as incoming Editor-in-Chief, but is something that came out of a survey of LMS Representatives over the summer months. Further changes exist with the (exceptional) insertion of a leaflet about legacies into paper copies of the *Newsletter*; and the expansion of the electronic version email. It is a lot to take, I know, but let's ring in some changes with the New Year.

Furthermore, are you the sort of reader who likes to read the issue from cover to cover? Would bad formatting annoy you, and poor spelling upset you? Then maybe you could help us. We are looking for some 'first readers' who would be happy to read the proofs before they go to press — and let us know of any inconsistencies spotted. *Newsletter* issues appear bi-monthly, the timescales are tight and the issues are quite lengthy — but your work would be much appreciated. If you could help, please let me know: newsletter.editor@lms.ac.uk.

Eleanor Lingham
Editor-in-Chief

LMS NEWS

LMS Council 2019–20

The results of the 2019 LMS Elections to Council and Nominating Committee were announced at the LMS Annual General Meeting on 29 November 2019. Council membership is as follows:

PRESIDENT:
Professor Jon Keating FRS (University of Oxford)

VICE-PRESIDENTS:
Professor Iain Gordon (University of Edinburgh)
Professor Cathy Hobbs (University of the West of England)

TREASURER:
Professor Robert Curtis (University of Birmingham)

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PROGRAMME SECRETARY:
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EDUCATION SECRETARY:
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Dr Mark McCartney (Ulster University)

MEMBERS-AT-LARGE OF COUNCIL:

*Professor Alexandre Borovik (University of Manchester)
*Professor Tara Brendle (University of Glasgow)
Professor Elaine Crooks (Swansea University)
Professor Andrew Dancer (University of Oxford) (re-elected to Council)
*Professor David E. Evans (University of Cardiff)
Dr Tony Gardiner (re-elected to Council)
Dr Frank Neumann (University of Leicester)
Professor Brita Nucinkis (Royal Holloway, University of London) (re-elected to Council)
Dr Richard Pinch
*Dr Mariya Ptashnyk (Heriot-Watt University)
*Professor Anne Taormina (University of Durham)

*Members elected in 2018 who are continuing with the second year of their two-year term.

Professor John Greenlees stood down at the AGM, having served as a Vice-President for ten years. The AGM also saw Professors Mark Chaplain and Gwyneth Stallard, and Dr Alina Vdovina, step down as Members-at-Large on Council. The LMS is grateful for their contributions, details of which can be found on page 7.

LMS NOMINATING COMMITTEE:

Also at the AGM, Professor Beatrice Pelloni (Heriot-Watt University) and Professor Mary Rees

(University of Liverpool) were elected to the Nominating Committee for three-year terms of office. Continuing members of the Nominating Committee are: Professor Kenneth Falconer (Chair), Professor I. David Abrahams, Professor Martin Mathieu, Professor H. Dugald Macpherson and Professor Elizabeth Winstanley. Council will also appoint a representative to the committee.

Incoming Officers and Members of Council

VICE-PRESIDENT:
Iain Gordon is a Professor of Mathematics at the University of Edinburgh who has also worked at the University of Glasgow and as an EU Research Fellow at the University of Bielefeld and the University of Antwerp. His research interests are representation theory and noncommutative algebra, and their connections with combinatorics and algebraic geometry. Professor Gordon's service to the LMS includes being a Member-at-Large on Council from 2005-2009, as well as a member of Programme Committee and Publications Committee during the same period. He was a member of the Research Meetings Committee from 2010-2012, Editor, Proceedings of the London Mathematical Society 2012-2015 and a member of the Prizes Committee from 2017 to date.

MEMBERS-AT-LARGE:
Elaine Crooks is a Professor in the Department of Mathematics at Swansea University. She has also worked at the University of Oxford, with short term postdoc appointments in Lausanne, Paris and Rome. Professor Crooks' research interests are nonlinear partial differential equations, singular limits of elliptic and parabolic systems, reaction-diffusion-convection systems and travelling waves, applications of PDE to biology, and geometric methods for image processing. She served as a member of the Editorial Advisory Board from 2009-2018.

Frank Neumann is an Associate Professor in Pure Mathematics in the School of Mathematics and Actuarial Science at the University of Leicester. His previous work has included positions at the Georg-August-Universität Göttingen and the CRM Barcelona. Research interests include algebraic topology, algebraic geometry and its interactions and in recent times homotopy theory and cohomology of algebraic stacks. Dr Neumann's LMS service includes acting as Chair, board member and mentor of the LMS Mentoring African Research in Mathematics (MARM) initiative, as the LMS Representative for the University of Leicester and as an organiser of regional joint research groups, regional meetings and a research school on Homotopy Theory and Arithmetic Geometry.

Richard Pinch is a retired civil servant who gained his PhD from the University of Oxford in 1983. From 1998 to 2018 Dr Pinch was a Mathematician at GCHQ; he has also served as Deputy Director of the Heilbronn Institute for Mathematical Research. Earlier in his career Richard Pinch lectured and conducted research at Cambridge University and the University of Glasgow. His research interests are computational number theory: primality test and Diophantine equation and algebraic combinatorics. Dr Pinch delivered the LMS Popular Lecture in 1994. He is currently Vice President (Professional Affairs and Industry) of the Institute of Mathematics and its Applications.

Access the LMS Book Series Online

The Society publishes two book series in partnership with Cambridge University Press: the Lecture Notes and the Student Texts.

If your institution has purchased electronic editions of titles in the series, you may be able to access the electronic versions of these books on the Cambridge Core platform. The titles in the series are listed at tinyurl.com/m6cm6ed for the Lecture Notes and tinyurl.com/wdjczpv for the Student Texts. If you do have access, this will be indicated by the word 'Access' (in green) and a tick mark.

LMS members are also entitled to a 25% discount when purchasing printed volumes in the series from Cambridge University Press.

Members of Council 2019–2020

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Cathy Hobbs
Vice President

Iain Gordon
Vice President

Rob Curtis
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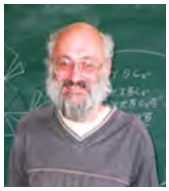

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Richard Pinch
Member-at-Large

Mariya Ptashnyk
Member-at-Large

Anne Taormina
Member-at-Large

Retiring Officers and Members of Council

CAROLINE SERIES
President 2017–2019

After serving as LMS President for two years, Professor Caroline Series FRS, handed over the badge of office at the AGM on 29 November 2019.

Since becoming LMS President, Professor Series has been a tireless advocate for the LMS, promoting the Society's events and activities and acting as a spokesperson for the Society's interests. She has been instrumental in taking forward changes that will ensure the robust governance of the Society and has put structures in place to help safeguard the Society's long-term financial stability.

During her period as President, Professor Series oversaw the conclusion of the review and update of the LMS Standing Orders (Charter, Statutes and By Laws), which will ensure the Society's governance structure is fit for purpose for the foreseeable future.

All surplus income from the LMS publications is used entirely to support mathematicians and mathematics research. This includes activities such as funding mathematics conferences, awarding mathematics research grants, giving prizes for mathematical accomplishments, and representing mathematics to government and national policymakers However, this income is potentially under threat due to the move towards Open Access. Thus, the long-term stability of the Society's programmes is increasingly likely to depend on endowments and gifts, while at the same time LMS Council is keen to further develop the volume and breadth of its activities.

To this end Professor Series has led the growth of the Society's fundraising activities and she has been instrumental in launching a Donations and Legacy page on the Society's website, making it easier for those who wish to donate money to the Society. There has also been the introduction of a new scheme, the 'De Morgan Friends', for those in a position to give £1,865 or more to the Society, which at the time of printing has already received its first two £1,865 donations. A new programme of De Morgan Dinners will also begin in 2020, which provide networking opportunities for the Society representatives with potential donors. Professor Series has taken the lead in establishing these programmes and activities as a means of encouraging opportunities for benefactors to work with the Society to help secure the future of its events and activities.

Professor Series has been an effective ambassador for the Society at a range of meetings in the UK and overseas including attending the Abel Prize ceremony in Oslo and the European Mathematical Society (EMS) Council meeting in Prague. Professor Series attended the EMS Presidents' Meeting in Maynooth, Ireland in 2018. She also led a delegation to the International Congress of Mathematicians (ICM) and the International Mathematical Union (IMU) General Assembly in Rio de Janeiro in 2018 and presided over the first Society Meeting ever held in the Southern Hemisphere. She has chaired Society Meetings, attended Society Regional Meetings and lectures across the UK, as well as the British Mathematical Colloquium (BMC) and joint meetings with the Institute of Mathematics and its Applications (IMA) and the Royal Statistical Society (RSS).

Professor Series has also been involved in the work of the Council for the Mathematical Sciences (CMS), in particular the continuing discussions on the implementation of recommendations stemming from the Bond Review. Professor Series is a member of the Big Mathematics Initiative (BMI) Strategic Committee, which was established to take this forward, and also set up an LMS Working Group to develop the Society's views to feed into this.

In May this year, Professor Series chaired the LMS 2019 Spring Reception, an event generously hosted by Dr Richard Golding, at which Dr Claire Craig, Chair of The BMI Strategic Committee spoke about the developments and aims of the BMI in taking forward the recommendations of the Bond Review, followed by a lively debate chaired by the President. This was a crucial networking opportunity for those in the Mathematical Sciences to meet with policy makers and those from industry and this year was once again a very successful event.

Under Professor Series' stewardship the LMS hosted the Abel Prize Committee in both 2018 and 2019. A first for the Society in 2019 was to co-host a live screening of the announcement of the winner of the Abel Prize in the Science Gallery at King's College, London. Professor Series played a major role in organising this prestigious event, convening a panel to discuss the life and work of the Abel Laureate of which she herself was a key participant.

This year marked the 21st Anniversary of the Society's purchase of its own building and establishment of its home at De Morgan House. Professor Series set up a Working Group to oversee the organisation of an event to mark this significant occasion in the Society's history and on 19 October 2019 the Society hosted the DeMorgan@21 event. This wonderful occasion was attended by LMS Officers, Council Members and staff, past and present, as well as LMS members and other friends of the Society. It was a fitting way to commemorate what was an historic and farsighted decision for the Society.

Following the sad demise of Sir Michael Atiyah, Professor Series has taken the lead in setting up the UK–Lebanon Atiyah Fellowship scheme in his memory. The scheme is operated in partnership with the Centre for Advanced Mathematical Sciences in the Lebanon and provides for an established UK based mathematician to visit the Lebanon for a period of up to 6 months, or alternatively for a mathematician from the Lebanon of any level, in particular promising advanced level students from the American University of Beirut, to visit the UK to further their study or research for a period of up to 12 months. One or more Fellowships may be awarded in any one academic year as funds allow. The Fellowships were officially launched during a live link with the American University of Beirut on 30 November 2019.

Also to commemorate the life and work of Sir Michael Atiyah, Professor Series has been the driving force behind organising the Atiyah Memorial Conference, which will take place at the Isaac Newton Institute (INI), Cambridge in September 2020. At the time of writing, the Clay Mathematics Institute had confirmed its intention to provide funding and some invited speakers had already agreed to give a talk.

Professor Series signed an historic Reciprocity Agreement on behalf of the Society with the Irish Mathematical Society (IMS). This enables LMS members who are not normally resident in the Republic of Ireland to join, or renew their subscription to, the Irish Mathematical Society and pay half the normal membership fee. LMS Members who are not normally resident in the United Kingdom, and who are also members of the Irish Mathematical Society, can now request a change in their membership type to Reciprocity member.

Professor Series also signed a contract with the International Mathematical Union (IMU) on behalf of the Society for the LMS to provide support for the administration of the International Mathematical Union IMU Breakout Graduate Fellowships Programme.

The Society would like to thank Professor Series for her dedicated service and for her enthusiasm and leadership during her two years in office and wishes her well for the future. At the AGM, Professor Series handed on the badge of Presidential Office to Professor Jon Keating, FRS.

JOHN GREENLEES
Vice-President 2009–2019

After 10 years Professor John Greenlees, Head of Mathematics, University of Warwick has stepped down as LMS Vice-President.

Professor Greenlees has given many years of service to the LMS dating back to 1994 as a member of the LMS Journals Editorial Board (1994–2004), Publications Committee (2004–09), BMC Scientific Committee (1999–2004), Prizes Committee (2009–10), LMS Council (2010–19), Research Policy Committee (2010–19, the past two years as Chair), and Women in Mathematics Committee (2010–19, Co-Chair 2013–18). During the Society's Anniversary celebrations in 2015 Professor Greenlees was a member of the 150th Anniversary Committee and chaired the Mathematics Promotion Unit/Joint Promotion of Mathematics Steering Group (2010–17).

Professor Greenlees has also served as a member of the EPSRC Mathematics Strategic Advisory Team (2004–06), Pure Mathematics RAE Panel F20 (2006–08), Mathematical Sciences REF Panel B10 (2011–14), and is currently the Deputy Chair of the Mathematical Sciences REF Panel B10 (2018–21).

Professor Greenlees has been influential in directing the Society's activities over a number of years. His wealth of experience and guidance has been invaluable on many issues across research policy, primarily as Chair of the LMS Research Policy Committee. He has been instrumental in providing input to consultations, working for both the LMS and the Council for the Mathematical Sciences (CMS), leading in areas of work such as *The Mathematical Sciences People Pipeline* report, the 2017 *Survey of Postdoctoral Researchers in the Mathematical Sciences in the UK* and a statement on the EU referendum outlining the CMS commitment to continuing EU collaboration. Through the Research Policy Committee he put in place a process to collect data from across a wide range of

areas in the Mathematical Sciences landscape, which has been identified by the Heads of Departments of Mathematical Sciences (HoDoMS) as being very important to the health of mathematics departments in the UK. He also co-led the development of the Mathematical Sciences Directory.

In other areas Professor Greenlees contributed to the 2018 LMS Publications Strategic Retreat and he also carried out a review of the Society's communications activities in 2013 which led to the redesign of the LMS Newsletter. His participation and ideas during the LMS 150th Anniversary year also led to among other things the production of *Impact 150: Stories of the Impact of Mathematics*, written by leading mathematicians based on mathematics done in the past 150 years.

He has been a prominent member of both the LMS Women in Mathematics Committee and the LMS Good Practice Scheme Steering Group, helping to produce both the 2013 report, *Advancing Women in Mathematics: Good Practice in UK University Departments*, which was launched in Parliament and the updated *National Benchmarking Survey 2017*. Professor Greenlees also played an influential role in the Women in Mathematics Committee being awarded the first Royal Society Athena Prize 2016, 'for introducing a broad range of initiatives in the field of mathematics resulting in a change of culture that has happened nationwide and leading the way in increasing the number of women in mathematics'.

Professor Greenlees' wide experience has benefited the Society and the mathematics community for many years and the LMS is extremely grateful to him for the support he has given to the Society over a long period.

MARK CHAPLAIN
Member-at-Large 2017–2019

Professor Chaplain, a former President of the Edinburgh Mathematical Society, has been a Member-at-Large on Council since 2017. In addition, he has served as Council representative on the Society's Publications Committee and as a member of the Research Grants Committee since 2018. The LMS thanks Professor Chaplain for his contributions to Council meetings, committee membership and continuing involvement with the work of the Society.

GWYNETH STALLARD
Member-at-Large 2013–2019

Professor Stallard has been a Member-at-Large on the Society's Council since 2014. A member of the Women in Mathematics Committee since 2003, she served as Chair from 2006 until stepping down from the Committee in 2015. The LMS representative from 2008-15 on the Athena Forum, an independent and expert voice on the issues of women's career progression and their representation in higher education and research, she was a member of the LMS team that was awarded the inaugural Royal Society Athena Prize in 2016. Professor Stallard has also sat on many of the Society's other committees: Prizes Committee (2010-2011), Nominating Committee (2007, 2014), Good Practice Steering Committee (from 2008), Programme Committee (from 2014) and Research Grants Committee, Early Career Research Committee and Personnel Committee (all from 2017). The Society is grateful to Professor Stallard for her commitment to achieving the aims of the Society over several years.

ALINA VDOVINA
Member-at-Large 2015–2019

Dr Vdovina has been a Member-at-Large on Council since 2015, also sitting as a member of the Research Grants Committee and the Society Lectures and Meetings Committee for four years. She was a member of the Organising Committee of the 58th British Mathematical Colloquium in Newcastle in 2006 and in 2012 organised the LMS-funded conference Beauville Surfaces and Groups. In 2013 she was an organiser of the LMS Durham Symposium. Dr Vdovina was a member of the LMS delegation to the ECM 2016 in Berlin and will also attend ECM 2020 in Slovenia on the Society's behalf. The Society would like to thank Dr Vdovina for her continuing work on LMS committees and ongoing support of various aspects of LMS work.

John Johnston
Society Communications Officer

Tributes to Sir Michael Atiyah: Atiyah UK–Lebanon Fellowships

Professor Sir Michael Atiyah, FRS, OM, who died on 11 January 2019, was a towering figure who dominated British and indeed international mathematics for more than half a century. Among his many roles and distinctions, he was LMS President 1974–76.

At its Retreat in February this year, Council discussed various possible ways in which the LMS might be able to honour and memorialise his name. Two actions were decided upon: firstly, the LMS took the initiative in proposing a major international conference on the 'Unity of Mathematics' in his honour. Sponsored jointly by the LMS, the Isaac Newton Institute, the Clay Mathematics Institute, the Heilbronn Institute and Oxford University, this will be held at the Newton Institute from 22 to 24 September 2020. More about the conference will appear in subsequent *Newsletters*.

Secondly, Council also wished to set up a more lasting memorial and is delighted to be able to announce the Atiyah UK–Lebanon Fellowships. These Fellowships are designed to facilitate a two-way visiting programme for mathematicians between the UK and Lebanon.

Born in England in 1929 to a Lebanese father and a Scottish mother, Michael Atiyah retained strong links with Lebanon throughout his life. He was made a Commander of the Order of Cedars in 1994, awarded an honorary degree from the American University of Beirut in 2004 and Lebanon Order of Merit (Gold) in 2005.

The Fellowship scheme will be operated in partnership with the Centre for Advanced Mathematical Sciences (CAMS) at the American University of Beirut (AUB). CAMS was founded in 1999 through the efforts of an international group of scientists with the primary goal of becoming the premier centre of excellence for research in the mathematical sciences in the Middle East, and itself owes much to the dedicated efforts of Sir Michael who was a frequent visitor.

The scheme provides for an established UK-based mathematician to visit Lebanon for a period of at least a week, up to a maximum of 6 months; or alternatively, for a mathematician from Lebanon of any level, in particular, promising advanced level students from the AUB, to visit the UK to further their study or research for a period of up to 12 months. One or more Fellowships may be awarded in any one academic year as funds allow.

The criterion for the award for mathematicians from the UK to visit Lebanon will be mathematical excellence, and willingness and ability to communicate and interact with mathematicians in the host institution. Award winners will be expected to deliver lectures during their tenure.

For mathematicians coming from Lebanon to the UK, the scheme provides travel and top up funds for a young mathematician to visit the UK as an Atiyah Fellow for purposes of study or research for a period of up to 12 months. Applicants must be at least at the level of an advanced MSc/PhD student in Mathematics or Mathematical Physics. Applications from mathematicians at a more advanced level to pursue research or collaborations in the UK are also welcome.

For further details and application forms, please visit lms.ac.uk/grants/atiyah-uk-lebanon-fellowships. The closing date for awards to be held in the academic year 2020–21 is 31 January 2020.

New LMS Editor of *Nonlinearity*

Professor Konstantin Khanin (University of Toronto) has been appointed new co-Editor-in-Chief of *Nonlinearity* from 2020, succeeding Carlangelo Liverani. He is joining Tasso Kaper (Boston University) in this new role after having contributed extensively as a member of the journal's editorial board.

Nonlinearity, founded in 1988 and jointly owned by the LMS and the Institute of Physics, covers the interdisciplinary nature of nonlinear science, featuring topics which range from mathematics to physics and engineering through to biological sciences.

The LMS would like to thank Professor Carlangelo Liverani for his valuable contributions and wishes the new co-Editor-in-Chief every success.

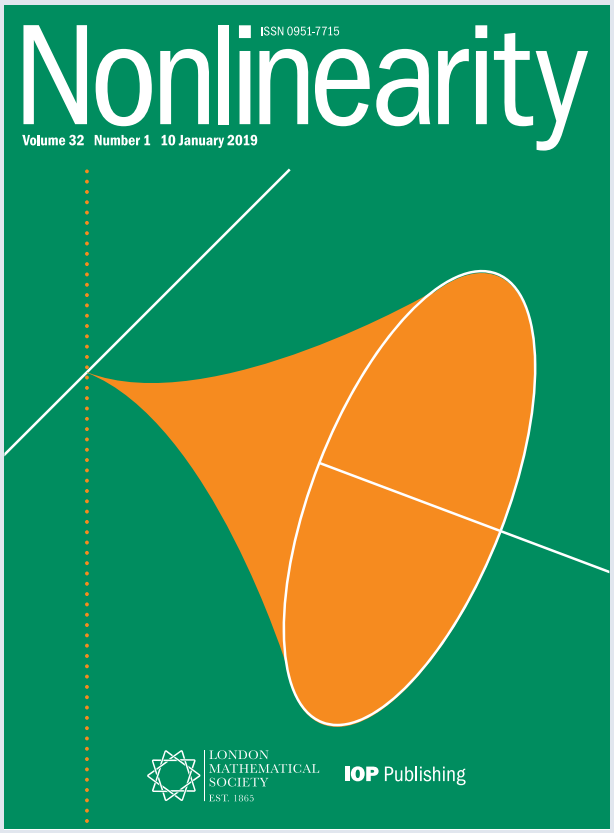
LMS members may freely access *Nonlinearity* via the membership subscription form or via member login on the LMS website (lms.ac.uk/user).

Nonlinearity

2018 JOURNAL IMPACT FACTOR 1.729

iopscience.org/non

Nonlinearity covers all areas of nonlinear phenomena, and addresses both the mathematics and physics communities.



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Editors-in-Chief

Tasso Kaper
Boston University, MA, USA

Konstantin Khanin
University of Toronto, Ontario, Canada
Newly appointed from 2020 by the London Mathematical Society

"Over the past 30 years *Nonlinearity* has published high-quality papers on nonlinear science understood in a very broad sense, often bridging between mathematics and physics. I am delighted to continue my work in *Nonlinearity*, and hope to contribute to the success of the journal in a new role of Editor-in-Chief.

Konstantin Khanin

To submit your work, or for more information, visit **iopscience.org/non**

LONDON MATHEMATICAL SOCIETY EST. 1865

IOP Publishing

Changes to the *Newsletter* Editorial Board

It is a pleasure to have this opportunity to record once again the Society's gratitude to Iain Moffatt for having given the new style *LMS Newsletter* such a flying start, and to welcome Eleanor Lingham as the new Editor-in-Chief.

However, this note is mainly to thank the other editors who are also leaving the Board this winter: Tomasz Brzezinski, Lucia Di Vizio, Bill Lionheart, Kitty Meeks, and Vicky Neale. It is no small thing to volunteer to be part of a project such as this, and the Society is grateful to all of them for their time, energy, humour, good judgement, and great ideas.

There are already two new members of the Board, Cathy Hobbs and David Chillingworth, who will be focussing on Society news. But there will be more: by the time this issue is published Council will have made at least three new appointments. Meanwhile, welcome to Cathy and David.

If any reader suddenly realises that they would really love to be a member of the Board, there is no need to panic! Further appointments will be made in due course.

Finally, thank you again to Tomasz, Lucia, Bill, Kitty, and Vicky.

Stephen Huggett
LMS General Secretary

Prize Awarded to Professor Cheryl Praeger

Professor Cheryl Praeger (University of Western Australia) was awarded the 2019 Prime Minister's Prize for Science. Professor Praeger received the Prize for contributions to group theory, permutation groups, combinatorics and the mathematics of symmetry. She received the Prize at a ceremony in October 2019 at Parliament House in Canberra.

The Prime Minister's Prizes for Science are annual Australian awards for outstanding achievements in scientific research, innovation, and teaching. The prizes have been awarded since 2000, when they replaced the Australia Prize for Science. The Prime Minister's Prize for Science is regarded as the national award for the advancement of knowledge through science and the winner receives AUD 250,000, an embossed solid gold medallion and lapel pin.

Professor Praeger has had a long and distinguished career in mathematics and received the award for her contributions to various areas of mathematics, including group theory, permutation groups, combinatorics and the mathematics of symmetry. Her expertise in group theory and combinatorial mathematics has underpinned advances in algebra research and computer cryptography.

Professor Praeger received her BSc and MSc degrees from the University of Queensland, and her PhD from the University of Oxford under the supervision of Peter Neumann. She has spent the majority of her career at the University of Western Australia where not only her research but her advocacy for mathematics in schools has been a major part of her work over the years.

Professor Praeger has received many honours and awards during her long career. She was the first female President of the Australian Mathematical Society (1992-94) and the Society now awards the Cheryl E Praeger Travel Awards to female mathematicians. Praeger became an Honorary Member of the London Mathematical Society in 2014 and the Society extends its congratulations to Professor Praeger on receiving the prestigious Prime Minister's Prize for Science.

Leaving a Legacy to the Society

Readers receiving a hard copy of the LMS *Newsletter* will notice that this issue includes a leaflet intended to assist anyone who is contemplating leaving a legacy to the Society.

Historically, we have been very fortunate to receive major donations and legacies, including those of Lord Rayleigh, Joseph Larmor, G. H. Hardy, Albrecht Fröhlich and Samuel Verblunsky. The Society is extremely grateful to anyone able to make a donation or remember the Society in their Will. The generosity of supporters contributes to an income which enables us to carry out vital work in support of the mathematics community, including research and conference grants, initiatives for students, early career researchers and teachers, and general promotion of mathematics. With income from publications potentially under threat owing to the move to Open Access, the long-term stability of the Society's programmes is increasingly likely to depend on endowments and gifts. At the same time, Council is keen to maintain, and ideally increase, its support for all its activities.

Notes on leaving a legacy to the Society in your Will can be found in the enclosed leaflet. In addition, the LMS website now includes more general information on how to make a donation to the Society: see lms.ac.uk/content/donations. We have also set up a new scheme, 'De Morgan Friends', for those in a position to make a donation of £1,865 or more; see lms.ac.uk/content/donations/demorgan.

If you would like to discuss any of this further, please contact development@lms.ac.uk.

Caroline Series
LMS Past President

Forthcoming LMS Events

The following events will take place in the next three months:

South West & South Wales Regional Meeting: 15 January, Bristol (tinyurl.com/ybzvrqr7)

Society Meeting at the Joint BMC-BAMC: 8 April, Glasgow (tinyurl.com/yarpowdo)

A full listing of forthcoming LMS events can be found on page 54.

OTHER NEWS

Speakers at ECM 2020

The organising committee of the 8th European Congress of Mathematics, to be held at Portorož, Slovenia (5–11 July 2020) are pleased to announce several new invited speakers: Professors Stanislav Smirnov, Kathryn Hess, Bojan Mohar, and Robin Wilson will deliver public lectures and Sir Martin Hairer KBE FRS will deliver the Hirzebruch lecture. See 8ecm.si/ for more news and to register for the Congress.

Tanja Labus
Office of the Organising Committee
8th European Congress of Mathematics

MATHEMATICS POLICY DIGEST

Brexit uncertainty harming UK science

According to new analysis by the Royal Society the UK's share of EU research funding has fallen by 28% since 2015. The UK's share of Horizon 2020 grant funding has fallen significantly, from its pre-referendum height. Brexit uncertainty is having a clear impact on the UK's ability to attract funding through the world's largest international R&D investment programme.

In 2015 — prior to the EU referendum — the UK secured 16% of the total Horizon 2020 grants (€1.49 billion) signed for in that year. In 2018, this figure had fallen to just over 11% (€1.06 billion). The full analysis is available at tinyurl.com/yhknqqv6.

Call to report gender bias in research publishing

The Royal Society of Chemistry (RSC) is calling on all scientific publishers to join it in transparently reporting gender statistics in a move towards equality in research and academia.

This follows a detailed analysis of over 700,000 research papers which showed that women face subtle barriers at each step of the publication process.

Findings from the report include:

- 35.8% of RSC authors were women
- 23.9% of RSC submissions are by female corresponding authors — this is the most senior position on a paper
- 18.4% of RSC citations have a corresponding author who is a woman
- Women are much less likely to be the sole author of a paper, representing 19.6% of single authorship papers submitted
- More articles from male lead authors are recommended for acceptance or minor revisions than from women
- Women are more likely to have an article rejected without peer review
- Male reviewers are more likely to recommend 'reject' for submissions from female lead authors than male lead authors.

The full report is available at tinyurl.com/yk35dmqs.

New President for IoP

Jonathan Flint CBE became the new President of the Institute of Physics in October 2019. He has been President-elect since October 2017 and will now serve for two years as President. More information is available at tinyurl.com/ygkab9a9.

Digest prepared by John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

Caucasian Mathematics Conference

The third Caucasian Mathematics Conference (CMC) successfully took place in Rostov-on-Don (Russia) 26-29 August 2019. With 120 mathematicians from 15 countries having registered, a large majority participated, with 15 plenary and 30 session talks delivered. The EMS Speaker Ragni Piene was the first woman to deliver a plenary lecture at CMC, and the ratio of women was above 20%. The Steering Committee plans to host the fourth CMC in Yerevan (Armenia) in August 2021, and the fifth in Azerbaijan, followed by Iran. CMC participants hope to act as an example of the possibilities for deepening cooperation between the countries of the region (euro-math-soc.eu/cmc/).

Committee for Developing Countries

The EMS-CDC established the Emerging Regional Centres of Excellence (*ERCE*) label to award institutes which are outstanding in their area of influence in research and education and which attract students from other regions and countries. The label is granted for four years with the possibility of renewal. The focus of this award is the education of students in the developing world to Masters level and possibly PhD. To train mathematicians from the least developed regions to Masters level, it is no longer necessary to send them to institutions in highly developed countries, since there are now more and more well-organised regional centres of excellence. The most recent Call for ERCE Proposals opened in November (nickpgill.github.io/emscdc/).

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

LMS Grant Schemes

The next closing date for research grant applications is 22 January 2020. Applications are invited for the following grants to be considered by the Research Grants Committee at its February 2020 meeting:

RESEARCH GRANTS

Conferences Grants (Scheme 1)
Grants of up to £7,000 are available to provide partial support for conferences held in the United Kingdom. Awards are made to support the travel, accommodation, subsistence and caring costs for principal speakers, UK-based research students and participants from Scheme 5 eligible countries.

Visits to the UK (Scheme 2)
Grants of up to £1,500 are available to provide partial support for a visitor to the UK, who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. It is expected the host institutions will contribute to the costs of the visitor.

Research in Pairs (Scheme 4)
For those mathematicians inviting a collaborator to the UK, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research.

Collaborations with Developing Countries (Scheme 5)
For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position.

Computer Science Small Grants (Scheme 7)
Grants of up to £1000 are available to support visits for collaborative research at the interface of Mathematics and Computer Science, either by the grant holder to another institution within the UK or abroad, or by a named mathematician from within the UK or abroad to the home base of the grant holder.

Research Workshop Grants
Grants of between £3,000 – £5,000 are available to provide support for Research Workshops held in the United Kingdom, the Isle of Man and the Channel Islands.

African Mathematics Millennium Science Initiative (AMMSI) Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms are available at ammsi.or.ke.

GRANTS FOR EARLY CAREER RESEARCHERS

The deadline is 22 February 2020 for applications for the following grants, to be considered by the Early Career Research Committee in March.

Postgraduate Research Conferences (Scheme 8)
Grants of up to £4,000 are available to provide partial support for conferences held in the United Kingdom, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants.

Celebrating new appointments (Scheme 9)
Grants of up to £600 are available to provide partial support for meetings held in the United Kingdom to celebrate the new appointment of a lecturer at a UK university.

Travel Grants for Early Career Researchers
Grants of up to £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences, or undertake research visits either in the UK or overseas.

LMS Research School Grants
Grants of up to £15,000 is available for a LMS Research School which provides training for research students in all contemporary areas of mathematics.

For full details of these grant schemes, and for information on how to submit an application form, visit lms.ac.uk/grants.

New LMS Grants for Teacher CPD: Teaching and Learning in HE

The LMS Grants for Teacher CPD: Teaching and Learning in HE is a new scheme intended to provide partial support of up to £500 for one-day workshops disseminating good practice in teaching undergraduate mathematics.

The scheme is intended to facilitate mathematical professional development to enable lecturers and teachers in HE institutions to:

- Develop their subject knowledge
- Engage in a deeper understanding of how to develop mathematical thinking
- Appreciate the interconnectivity of mathematical topics
- Update themselves on mathematics curriculum reform
- Use technology when and where appropriate.

See the full criteria and download an application form at tinyurl.com/u9g6gsd. There are four rounds of awards per year and the deadline for the next round is 30 January 2020. Send completed applications to Katherine Wright, Society and Research Officer: education@lms.ac.uk.

LMS Prospects in Mathematics Meeting 2021: Call for Expressions of Interest

UK departments are invited to submit expressions of interest to host the LMS Prospects in Mathematics Meeting 2021 to the Prospects in Mathematics Meeting Steering Group by 1 March 2020.

Up to £7,000 is available to support the annual two-day events (usually taking place in September) for Finalist Mathematics undergraduates who are considering applying for a PhD after they have completed their current studies. This includes funding to cover fares and accommodation for up to 50 students, travel and accommodation for speakers and subsistence for participants including a social event.

LMS Prospects in Mathematics Meetings should feature speakers from a wide range of mathematical fields, and from across the UK, who discuss their current research and what opportunities are available to prospective PhD students.

Prospective organisers should send an expression of interest (max. one A4 side in length) with the following details:

- Department's confirmation of support to host the LMS Prospects in Mathematics Meeting.
- Reasons to host the LMS Prospects in Mathematics Meeting.
- A provisional list of speakers who are representative of the UK research landscape both geographically and scientifically.
- Speakers from under-represented groups should be included and women speakers should account for at least 40% of the invited speakers.
- Confirmation that prospective organisers have read and understood the terms and conditions in the Guidelines for Organisers (available from tinyurl.com/y9yn2ryo)
- Willingness to attend an upcoming LMS Prospects in Mathematics Meeting in Bath from 10–11 September 2020 to get an idea of the event.

For further details, visit tinyurl.com/y9yn2ryo.

LMS Early Career Fellowships: 2019–20 Awards

To support early career mathematicians in the transition between PhD and a postdoctoral position, the LMS offers up to eight fellowships, of between three and six months, to those who have recently or will shortly receive their PhD. The award will be calculated at £1,200 per month plus a travel allowance. The fellowships may be held at one or more institutions but not normally at the institution where the fellow received their PhD. The 2019–20 round is now open to applications (deadline: 14 February 2020). For further details, visit tinyurl.com/y7npy2q7. Contact Elizabeth Fisher (lms.ecf@lms.ac.uk or 020 7291 9973) with any queries.

LMS Invited Lectures Series 2021: Call for Proposals

The annual LMS Invited Lectures Series consists of meetings held in the UK at which a single speaker gives a course of about ten expository lectures, examining some subject in depth, over a

five-day period (Monday to Friday) during a University vacation. The meetings are residential and open to all interested.

The LMS Invited Lecturer is offered a £1,250 honorarium for giving the course plus full expenses. A £4,000 grant is given to the host department to support attendance at the lectures.

Proposals for the Invited Lectures 2021
Any member who would like to suggest a topic and lecturer and be prepared to organise the meeting at their own institution, or a suitable conference centre, can submit a proposal. For further details, visit tinyurl.com/y98espkj. The deadline for proposals is 1 February 2020.

LMS Invited Lecturer 2020
The LMS Invited Lecture Series 2020 on fractional calculus and fractional stochastic calculus, including rough-paths and applications, will be given by Professor Yuliya Mishura (University of Kyiv) at Brunel University, London, from 30 March – 3 April 2020.

Recent previous Invited Lecturers have included S. Asmussen (Aarhus University, 2019), A. Owen (Stanford University, 2018) and J. Agler (UC San Diego, 2017).

Enquiries about the LMS Invited Lectures may be addressed to the Chair of the Society Lectures and Meetings Committee, Brita Nucinkis (lmsmeetings@lms.ac.uk).

PROMYS Europe 2020 Call for Applications

PROMYS Europe, a challenging six-week residential mathematics summer programme at the University of Oxford is seeking applications from pre-university students from across Europe (plus all countries adjacent to the Mediterranean) who show unusual readiness to think deeply about mathematics, as well as from undergraduate students who would like to work with them as counsellors. PROMYS Europe is a partnership of Wadham College and the Mathematical Institute at the University of Oxford, the Clay Mathematics Institute, and PROMYS (Program in Mathematics for Young Scientists, founded in Boston in 1989).

PROMYS Europe is designed to encourage mathematically ambitious students who are at least 16 to explore the creative world of mathematics. Participants tackle fundamental mathematical questions within a richly stimulating and supportive community of fellow first-time students, returning students, undergraduate counsellors, research mentors, faculty, and visiting mathematicians. Counsellors receive a stipend, and are given free meals and accommodation for the duration of the programme.

Applications for counsellors and students open in January and are available on the PROMYS Europe website www.promys-europe.org. The closing date for counsellor applications is 9 February. The closing date for pre-university student applications is 15 March, and students will need to allow enough time before the deadline to tackle the application problems. PROMYS Europe 2020 will run from 12 July to 22 August at the University of Oxford.

LMS Hardy Lectureship 2020

The LMS is pleased to announce that Professor Peter Sarnak FRS, of Princeton University and the IAS, will be the LMS Hardy Fellow 2020. He will give a series of lectures in the UK during the period 15 June to 3 July 2020.

The Hardy Lectureship was founded in 1967 in memory of G.H. Hardy and in recognition of an outstanding contribution to both mathematics and to the Society. The Hardy Lectureship is a lecture tour of the UK by a mathematician with a high reputation in research.

Professor Sarnak works in number theory and related areas. He has made major contributions to several areas, in particular pioneering developments in arithmetic statistics, quantum ergodicity, and thin groups. His work has also had major impact outside of number theory, such as his work on expander graphs.

The titles of his lectures are:

- *Integral Quadratic Forms and Applications*
- *Integer Points on Affine Cubic Surfaces*
- *Applications of Points on Sub Varieties of Tori*
- *Möbius Randomness and Dynamics*
- *The Topologies of Random Real Algebraic Hypersurfaces*

Calls for Mathematics Departments

Any university which would like to invite this year's Hardy Lecturer to their own institution or

a suitable conference centre to give a lecture can contact LMSMeetings@lms.ac.uk for consideration by the LMS Society Lectures and Meetings (SLAM) Committee.

The local organisers will be expected to:

- Make local arrangements including accommodation
- Liaise with the LMS to prepare posters and publicity
- Publicise the event as widely as possible
- Publicise the event as the LMS Hardy Lectureship unless otherwise agreed with the Society
- Utilise the funds of £100 contributed by the LMS and given to the host department to hold a dinner for the Hardy Lecturer
- Submit invoices to the LMS within three months of the event.

The deadline for department submissions is 14 February 2020.

Recent previous Invited Lecturers include Lauren Williams (UC Berkeley, 2018), Jacob Lurie (Harvard, 2016) and Nalini Joshi (Sydney, 2015).

For further details visit the website: tinyurl.com/h5ql729. Enquiries about the Hardy Lectureship may be addressed to Brita Nucinkis, Chair of the LMS Society Lectures and Meetings Committee (lmsmeetings@lms.ac.uk).

Reminders

LMS Prizes: call for nominations
Details at tinyurl.com/prizes2020. Deadline for nominations: 31 January 2020.

Bachelier Prize 2020: call for nominations
Details at tinyurl.com/bachelier2020. Deadline for nominations: 31 January 2020.

Zeeman Medal 2020: call for nominations
Details at tinyurl.com/zeeman2020. Deadline for nominations: 28 February 2020.

VISITS

Visit of Amarjit Budhiraja

Professor Amarjit Budhiraja (University of North Carolina at Chapel Hill) will visit the UK from 1 January to 31 March 2020. His research interests include theory of large deviations, stochastic control, stochastic networks, stochastic partial differential equations, random graphs and nonlinear filtering. He will give lectures at the University of Cambridge, University of Oxford, and Imperial College. For further details contact Dan Crisan (d.crisan@imperial.ac.uk). Supported by an LMS Scheme 2 grant.

Visit of Igor Erovenko

Professor Igor Erovenko (University of North Carolina at Greensboro) will visit City University of London from 11 to 17 May 2020. Igor's research interests lie in mathematical biology, including evolutionary game theory, game-theoretic models of infectious diseases, and theoretical ecology. He will give a seminar *A Primer on Evolutionary Graph Theory: How Network Topology Determines the Evolution of Cooperation on Multiplayer Networks*. For further details contact Mark Broom (Mark.Broom@city.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Patricio Gallardo

Dr Patricio Gallardo (University of California at Riverside) will visit the University of Essex from 1 to 21 March 2020. His research interests include moduli spaces of algebraic varieties and computational algebraic geometry. As part of his visit, he will give talks at Imperial College London (9 March) and the University of Essex (12 March). Contact jesus.martinez-garcia@essex.ac.uk for further details. The visit is supported by an LMS Scheme 4 grant.

Visit of Ching Hung Lam

Dr Ching Hung Lam (Academia Sinica, Taiwan) will visit the UK from 17 May to 6 June 2020. He will give talks at the University of Birmingham, Imperial College and University of Bristol. Dr Lam works in vertex operator algebras and will collaborate on a related topic of axial algebras. For further details contact Sergey Shpectorov (s.shpectorov@bham.ac.uk). Supported by an LMS Scheme 2 grant.

Visit of Ron Lifshitz

Professor Ron Lifshitz (Tel Aviv University) will visit the UK from 9 to 23 February 2020 and will deliver colloquia at Universities of Oxford, Loughborough and Leeds. His research focus is in nanomechanical systems, nonlinear dynamics and quasicrystals. He will also participate in the EPSRC-funded meeting *Theories and Methods for Soft Matter Quasicrystals* at Loughborough. For further details contact Priya Subramanian (priya.subramanian@maths.ox.ac.uk). Supported by an LMS Scheme 2 grant.

Visit of Maxim Pavlov

Dr Maxim Pavlov (Lebedev Physical Institute, Russian Academy of Sciences) will visit Loughborough University from 23 to 29 January 2020. His research interests include integrable systems, Hamiltonian and Lagrangian formalism, theory of Frobenius manifolds and Whitham averaging theory. For further information email Evgeny Ferapontov (E.V.Ferapontov@lboro.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Michael Plank

Professor Michael Plank (Canterbury, New Zealand) will visit the University of Essex from 18 to 23 May 2020. His research is at the interface of applied mathematics and biology, spanning scales from intracellular signalling and collective cell behaviour to large ecosystem dynamics and ecological and social networks. He will give a seminar on his recent research work, with specific focus on random walk models of movement. For further details contact Edward Codling (ecodling@essex.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Vladislav Sidorenko

Professor Vladislav Sidorenko (Keldysh Institute for Applied Mathematics, Moscow) will visit the UK from 8 to 21 March 2020. During his visit, he will give lectures at Imperial College, University of Cambridge and Loughborough University. His research is related to development and application of the perturbation theory for slow-fast dynamical systems in celestial mechanics. For further details contact Anatoly Neishtadt (a.neishtadt@lboro.ac.uk). Supported by an LMS Scheme 2 grant.

Visit of Roberto Svaldi

Dr Roberto Svaldi (Ecole Polytechnique Fédérale de Lausanne) will visit King's College London from 2 to 15 February 2020. His research interests focus on the applications of the Minimal Model Program to a wide range of questions in geometry, including the study of Calabi–Yau varieties and holomorphic foliations. For further information email Calum Spicer (calum.spicer@kcl.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Hans-Olav Tylli

Dr Hans-Olav Tylli (University of Helsinki) will visit the UK from 5 to 15 May 2020. His expertise is in the geometry of Banach spaces and operator theory. During most of his visit, Dr Tylli will be based at Lancaster University. He will give lectures at the University of Cambridge on 6 May, Queen's University Belfast on 8 May and Lancaster University on 13 May. For further details contact Niels Laustsen (n.laustsen@lancaster.ac.uk). The visit is supported by an LMS Scheme 2 grant.

Visit of Zihui Zhao

Dr Zihui Zhao (University of Chicago) will visit the UK for a month during April and May 2020. Dr Zhao's research interests are in partial differential equations, geometric measure theory and harmonic analysis. During his visit he will give research seminars at Cambridge, UCL, Oxford and University of Leeds. For further details contact Neshan Wickramasekera (ngw24@dpmms.cam.ac.uk). The visit is supported in part by an LMS Scheme 2 grant.

LONDON MATHEMATICAL SOCIETY
EST. 1865

Heilbronn Institute for Mathematical Research

Graph Packing

LMS Research School

Eastbourne; 19-25 April 2020

Organisers: Peter Allen, Julia Böttcher and Jozef Skokan (London School of Economics)

The application of random processes, the regularity method, and the absorbing method are three powerful modern techniques in Extremal Combinatorics which featured in many significant advances in the field over the last 20 years. One topic that brings all these three approaches together is that of graph and hypergraph packing. Packing problems study conditions under which copies of different graphs can be embedded edge-disjointly into a given host graph. There are applications of this in such diverse areas as information theory, computational complexity, computational biology, experiment design, and the theory of combinatorial games. With the help of the above-mentioned techniques, celebrated breakthroughs were recently possible on a number of long-standing and difficult questions in this area.

The purpose of this Research School is to introduce young researchers to these problems, results, and techniques, and to explore open questions in the field and in related research areas.

Apply at https://www.surveymonkey.co.uk/r/RS49AppInForm by Friday 31 January 2020. Research students, post-docs and those working in industry are invited to apply.

For further information please visit: http://personal.lse.ac.uk/boettche/ResearchSchool/

LSE

HEILBRONN DOCTORAL TRAINING PARTNERSHIP
PhD in Mathematics

Heilbronn Institute for Mathematical Research

The Heilbronn Doctoral Training Partnership invites applications for fully-funded PhD studentships in association with the Universities of Bristol, Manchester and Oxford. We are seeking applicants with research interests in Discrete Mathematics, interpreted in its broadest sense, which includes most areas of Pure Mathematics, Computational Statistics, Data Science, Probability and Quantum Information.

In addition to undertaking cutting-edge research in one of the partner universities, students on this programme will have the opportunity to spend nine weeks each summer (in years 1-3) at the Heilbronn Institute for Mathematical Research (HIMR), where they will contribute to the classified research activities of the institute. However, this is not a requirement of the studentship.

Successful candidates who wish to work at the Heilbronn Institute must satisfy vetting in order to engage with the classified research at HIMR and UK resident UK nationals will normally be able to meet this condition.

Students from traditionally under-represented groups are strongly encouraged to apply.

For further information about HIMR and this new initiative, together with details on how to apply, please visit our website: https://heilbronn.ac.uk/postgrad-students/

LMS Library at UCL: Member Benefit

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Members of the Society can register as users of the UCL Library, where the London Mathematical Society Library is held. The LMS Library contains a collection of:

- periodicals published by other mathematical societies which are received in exchange for the Society's publications
- copies of books and journals published by the Society
- items acquired by the Society as review copies or gifts.

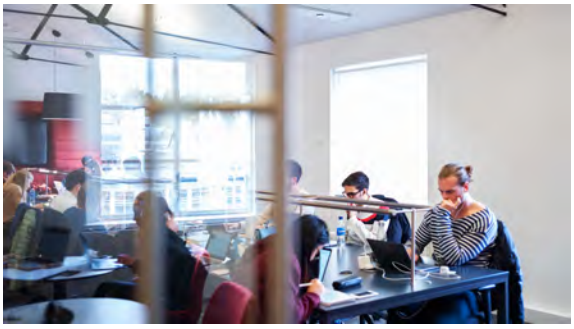
UCL Library Privileges

- Use of all the material available in the reading rooms and stores of the UCL family of libraries.
- Borrowing up to ten items at any one time.
- Placing up to three concurrent reservations on material already on loan.
- Borrowing books by post without service charge (costs for returning the books must be covered by the user).
- Access to MathSciNet and specific electronic journals from designated terminals in the Science Library.
- Use of the *Explore* access points to search for and view electronic publications and save single copies of articles (no more than one article per journal issue) for your own personal use. You can save articles to standard USB sticks, but note that USB sticks containing encrypted software do not work on the *Explore* access points.
- Use of photocopying facilities at UCL libraries (charged at the same rate as UCL staff).
- Rapid photocopying service by post.

For licensing reasons, remote electronic access to journals and articles is not included in UCL Library membership. To check the listings of electronic journals available to visitors, before your visit to the Library, use Explore (tinyurl.com/yb7wtqs6).

Registering and Renewing

For details on how to register/renew your UCL Library card visit tinyurl.com/ybxlzavw. No charge is made for the initial registration or for renewing expired library cards or cards which are within one calendar month of expiring. Library cards are valid for 12 months from date of issue and should be renewed each year.



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Visiting the Library

- Opening hours can be found at tinyurl.com/ybja2tmu.
- During the weekends and evenings, the Library is open principally to offer book loans and to handle related queries. For all other queries contact a member of staff during office hours at tinyurl.com/ycqjzxjt.
- Check seat availability in the reading rooms and computer cluster at the Science Library at tinyurl.com/y7b8vu5v.

For further information about the Society's Library visit lms.ac.uk/library/lms-library.

LMS Council Diary — A Personal View

After a long summer break, Council met again at De Morgan House on Friday, 18 October. As usual, the meeting began with an update on the President's activities since the last Council meeting, which included meetings of the All-Party Parliamentary and Scientific Committee and the Strategic Committee of the Big Mathematics Initiative.

Council then heard an update on the proposed Atiyah Fellowship for exchanges between Lebanese and UK mathematicians. These fellowships were approved and will be announced by the President as part of the 20th anniversary celebrations of the Centre for Advanced Mathematical Sciences at the American University of Beirut on 30 November, with a short address on Sir Michael Atiyah being given by Graeme Segal. The announcement and talk will be made from De Morgan House and live-streamed directly to the celebrations in Beirut.

We discussed, probably for the final time, the Standing Orders Review and approved the proposed Standing Orders amendments to be put to a vote of the membership at the Annual General Meeting. The President, on behalf of Council members, thanked the General Secretary and the Standing Orders Review Group for their efforts in getting the Society to this stage.

The Treasurer introduced the Trustees' Report and Annual Accounts. Council agreed to recommend this report, including the Annual Accounts for 2018–19, to the Annual General Meeting in November. The Treasurer further reported on the Cecil King Scholarships, and proposed that the award be increased from £5k to £6k annually, and that two awards might be made in each year, to show the breadth of mathematics. This was approved by Council.

At the last Council meeting we had discussed the Undergraduate Research Bursaries, and decided to defer the matter to this October meeting. In the meantime, the Programme Secretary and Tara Brendle, Member-at-Large, had each produced a paper with a proposal. As a result Council confirmed its view that these bursaries were research training, not employment, but decided that the weekly rate should be increased to £215 to reflect the amount paid in the National Living Wage scheme. Further, the Bursaries should be increased annually in line with inflation from next year onwards.

We also heard updates from Vice-President Hobbs on the draft communications review, from the Publications Secretary that the expected consultation document on Open Access from UKRI would be delayed, as well as annual reports from several committees.

Brita Nucinkis
Member-at-Large of Council

Longstanding LMS Members

Over 75 years of membership (elected in 1943): Freeman J. Dyson

Over 70 years of membership (elected in 1946–50): David Borwein, Bernard Fishel, Walter K. Hayman, Eric L. Huppert, Godfrey L. Isaacs

Over 65 years of membership (elected in 1951–55): John C. Amson , J. Vernon Armitage, Daniel E. Cohen, Hanafi K. Farahat, Ioan M. James, Francis Rayner, John R. Ringrose, Paul A. Samet, S. James Taylor, G.E. Wall

Over 60 years of membership (elected in 1956–60): Bryan Birch, John F. Bowers, Aldric L. Brown, Ronald Brown, David A. Burgess, Roger W. Carter, Gearoid De Barra, David E. Edmunds, David A. Edwards, Richard K. Guy, A. Howard M. Hoare, Roland F. Hoskins, Glenys Ingram, Donald Keedwell, Lionel W. Longdon, I.G. Macdonald, Joseph F. Manogue, Ismail J. Mohamed, David Monk, Alun O. Morris, Michael F. Newman, Roger Penrose, Roy L. Perry, Dennis C. Russell, Eira J. Scourfield, Dona Strauss, Ronald F. Turner-Smith, Alan West

Over 55 years of membership (elected in 1961–65): J. Clifford Ault, Patrick D. Barry, John C.R. Batty, Benjamin Baumslag, Alan F. Beardon, Homer Bechtell, Thomas S. Blyth, William Brown, John H.E. Cohn, Bruce D. Craven, Hallard T. Croft, Charles W. Curtis, Roy O. Davies, Ian M.S. Dey, Vlastimil Dlab, M.M. Dodson, J. Keith Dugdale, Peter L. Duren, L.C. Eggan, David Epstein, John Erdos, James Edward A. Evans, W. Desmond Evans, O.C. Ezeilo, Matthew P. Gaffney, Cyril F. Gardiner, David J.H. Garling, Peter Giblin, Robin E. Harte, Philip Heywood, Keith E. Hirst, Otto H. Kegel, John F.C. Kingman, J. David Knowles, David G. Larman, Ronald Ledgard, Peter G. Lowe, Malcolm T. McGregor, Hugh Morton, Albert A Mullin, Roy Nelson, Peter M. Neumann, Alan R. Pears, John

E. Peters, Frederick C. Piper, John S. Pym, George A. Reid, Frank Rhodes, Joseph B. Roberts, Stewart A. Robertson, Derek J.S. Robinson, W. John Robinson, H. Peter Rogosinski, James Edward Roseblade, John W. Rutter, Arthur D. Sands, Bruce L.R. Shawyer, Abe Sklar, Brian F. Steer, Anthony C. Thompson, Graham F. Vincent-Smith, Grant Walker, C. Terence C. Wall, Eric W. Wallace, John F. Watters, Alfred Weinmann, David J. White, Joyce E. Whittington, Sheila O. Williams

Over 50 years of membership (elected in 1966–69): R.B.J.T. Allenby, Mark A. Armstrong, Irene A. Ault, Anthony D. Barnard, Nicholas H. Bingham, M.C. Bramwell, Roger M. Bryant, Peter J. Bushell, Allan G.R. Calder, Munibur R. Chowdhury, Donald J. Collins, Michael J. Collins, Harold Garth Dales, P. Laurie Davies, Richard Delanghe, M.A.H. Dempster, Peter Dixon, Patrick Dolan, John Duncan, Martin J. Dunwoody, Roger H. Dye, Barry G. Eke, K. David Elworthy, Roger A. Fenn, Colin R. Fletcher, James W.M. Ford, John B. Fountain, Charles Goldie, Wilfrid A. Hodges, Jonathan Hodgson, Terence H. Jackson, Graham J.O. Jameson, David L. Johnson, Michael E. Keating, Thomas J. Laffey, E. Christopher Lance, Earl E. Lazerson, David W. Lewis, W.B. Raymond Lickorish, Bernard L. Luffman, Bob Margolis, John McKay, Peter McMullen, Ian M. Michael, William Moran, Michael Moss, Kung-Fu Ng, Peter J. Nicholls, David R. Page, Fredos Papangelou, Jon V. Pepper, Oliver Pretzel, Hilary A. Priestley, John F. Rennison, Colin P. Rourke, Keith Rowlands, Peter Rowlinson, Stephan M. Rudolfer, Philip Samuels, Rodney Sharp, John Silvester, David Singerman, Nelson M. Stephens, Bill Stephenson, W. Brian Stewart, Gabrielle A. Stoy, Anthony E. Stratton, R.F. Streater, David Tall, Brian Thorpe, David Tipple, Rabe R. von Randow, Martin Antony Walker, Jeffrey R.L. Webb, Bertram Wehrfritz, Dominic J.A. Welsh, Christopher Wensley, Thomas A. Whitelaw, Christopher M. Williams, Geoffrey V. Wood, Douglas R. Woodall, J.D.M. Wright, Abraham Zaks

50 years of membership (elected in 1970): Roger C. Baker, Earl R. Berkson, David A. Brannan, F. Trevor Brawn, John D. Dixon, Alastair Gillespie, Paul R. Goodey, T. John A. Haight, A. Geoffrey Howson, Monsur A. Kenku, Peter Kopp, Charles Leedham-Green, John H. Loxton, Vassilis Mavron, Adam J. Ostaszewski, J. Christopher Robson, Harvey Rose, David B. Singmaster, Linda R. Sons, Elizabeth Taylor, R. Kenneth Thomas, R.C. Vaughan FRS, Michael R. Vaughan-Lee.

Membership of the London Mathematical Society

The standing and usefulness of the Society depends upon the support of a strong membership, to provide the resources, expertise and participation in the running of the Society to support its many activities in publishing, grant-giving, conferences, public policy, influencing government, and mathematics education in schools. The Society's Council therefore hopes that all mathematicians on the staff of UK universities and other similar institutions will support mathematical research by joining the Society. It also very much encourages applications from mathematicians of comparable standing who are working or have worked in other occupations.

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REPORTS OF LMS ACTIVITIES

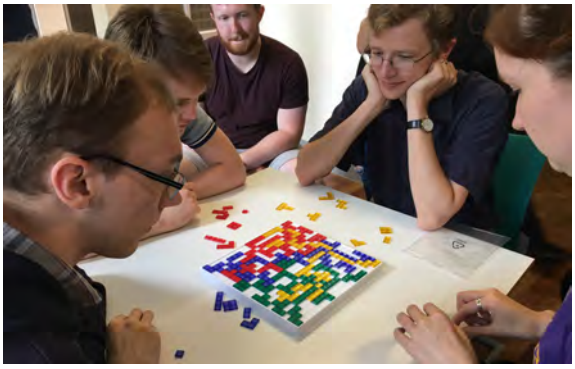
Report: LMS Undergraduate Summer School

Do you remember your first experience of research-level mathematics? This summer, 43 undergraduate students from around the UK descended on Leeds to have their first taste of mathematics beyond the undergraduate curriculum at the LMS-funded Undergraduate Summer School.

The students lucky enough to attend the School were high-achieving second- and third-years, selected from mathematics departments around the country. They were as diverse as the undergraduates at any UK university: while the majority lived in the UK, a good number were international students (from as far afield as China) studying at UK universities. Some were mature students who had switched to mathematics after successful careers in other disciplines.

The two-week programme consisted of six intense five-hour courses on advanced mathematical topics, complemented by nine colloquium-style talks. All were delivered by visiting speakers, who had generously given up their time to prepare materials and deliver lectures. The lecture courses opened with Summer School veteran Paul Sutcliffe's exposition of topological solitons in magnetic materials. This was followed by Marianne Johnson's course on tropical algebra and combinatorics, which took students from the basics of the subject to discussing theorems proved within the last year. The first week ended with Norbert Peyerimhoff giving students their first taste of curvature, in the context of graphs. In the second week, students learned in Tomas Johansson's Inverse Problems course that seemingly simple tasks such as solving linear equations can lead to surprising and unexpected problems. Sira Gratz discussed appearances of the ubiquitous and mysterious letters A, D and E (which might stand for "Amazing Diagrams Everywhere", or perhaps "Aliens Do Exist"). Finally, all available pairs of scissors and rolls of tape were put to good use in Saul Schleimer's hands-on course on low-dimensional topology.

The colloquia covered everything from the foundations of the subject (Mirna Dzamonja *Can we answer every question in mathematics?* and Nick Bingham *Two pillars of probability*) through to open problems at the forefront of mathematics (Peter Cameron *Problems around groups, graphs and semigroups* and *The random graph*). Some gave students a taste of mathematical beauty (Gwyneth Stallard *The beauty of fractals* and Martin Bridson *Symmetries, curved universes and unsolvable problems*). Talks covered the full spectrum of pure (Sanju Velani *Aspects of metric number theory*) and applied mathematics (Priya Subramanian *A brief history and minimal recipes for quasicrystals* and Mariya Ptashnyk *Multiscale modelling and analysis of biological systems*).



Despite the busy schedule, the students still had energy left for leisure activities at the end of each day. The group discovered a common passion for board games, and organised a tournament of the popular game Blokus (the organiser finished a respectable seventh). At the weekend there was time to get away from Leeds to explore God's Own County, with the group enjoying an invigorating walk from Coniston in the Yorkshire Dales. At the end of two weeks everyone returned home, with eyes opened to the wider world of mathematics and long-lasting friendships formed.

Derek Harland
University of Leeds

Report: Daphne Jackson Trust 2019 Conference

Have you heard the one about the mathematician, the chemist, the physicist, the biologist and the engineer? If not, you should have been at The Daphne Jackson Trust 2019 Conference! Partly funded by the London Mathematical Society, the conference attracted the best and brightest minds across STEM subjects to inspire and energise those returning to research.

The Daphne Jackson Trust is the UK's only charity dedicated to scientists returning to their research career following a break due to family, caring or health reasons. The Trust offers flexible, part-time, fellowships in universities and research Institutes in the UK and ROI. Fellows undertake a research project and an individually tailored retraining programme preparing them for a successful return to the research workforce. Founded in 1992 in memory of Professor Daphne Jackson (renowned nuclear physicist and the first female professor of physics in the UK), the charity has supported over 380 returners to research.

Many researchers don't necessarily take traditional career paths and sometimes 'life' happens... For those who took a career break — perhaps for having a family, health reasons or caring for an ill relative — it can be incredibly difficult to return to the research career you love. On paper, it might look like you've lost your competitive edge and fallen behind your peers. But the Daphne Jackson Trust believes that shouldn't stop you fulfilling your potential. The charity awards Fellowships to those wanting to return to research so that they can re-acquaint themselves with the very latest in their field. Many go on to win further funding and achieve brilliant things across STEM subjects.

The Daphne Jackson Trust's fifth biennial conference took place on 16 and 17 October 2019. It examined what their Fellowships are, why they are needed and how they are managed. With presentations from leaders across the STEM subjects, the conference demonstrated how Daphne Jackson Trust Fellowships changed the lives of their recipients.

Two of its fellows are sparkling mathematicians. Dr John Harvey, whose Fellowship is sponsored by Swansea University and the EPSRC, is working at the interface between geometry and probability theory, tackling questions inspired by data science. John has a PhD in geometry from the University of Notre Dame in the U.S. He held a post-doctoral position at the University of Münster in Germany and is now returning to research after a three-year break. Dr Hanan Dreiwi is undertaking her Fellowship at the University of Leeds. She is applying her mathematical skills to help those with psoriasis. Her research aims to develop mathematical, statistical and computational approaches to model asymmetric cell division — a brilliant example of a multidisciplinary approach that may one day help those plagued by this potentially debilitating skin condition.

The conference brought together current and former Fellows to showcase their original research from across the STEM spectrum. With posters and prizes, quizzes and competitions, the day's informal approach befittingly demonstrated the brilliance of those who may not have had the chance to return to research without the Trust's help.

The day finished up with something of a Dame bonanza! Chaired by the Trust's Patron, Dame Athene Donald, the three past and present Presidents from the Institute of Physics, Royal Society of Biology and Royal Society of Chemistry — Dame Julia Goodfellow, Dame Julia Higgins and Dame Carol Robinson — discussed their careers, challenges and future plans. An inspiring end to a very special day.

Dr Andy Clempson
Daphne Jackson Trust

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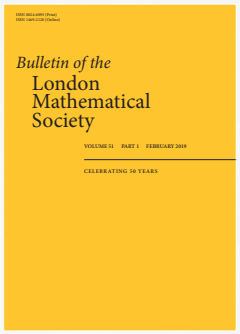

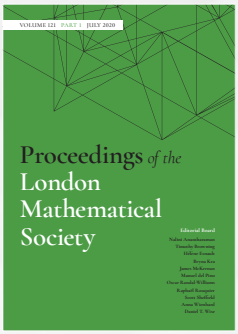

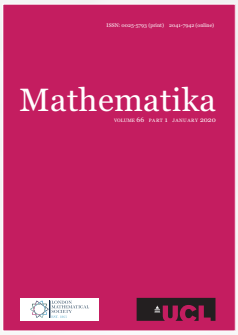

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Some Historical Developments of the Theory of Transcendence and Algebraic Independence

GWLADYS FERNANDES

Qu'est la majesté de ce qui finit, auprès des départs titubants, des désordres de l'aurore?[1]. Let this question from the French writer Colette guide us to walk through the promising beginnings, fascinating evolutions and results, challenging open problems and perspectives of an exciting theory that is far from revealing all its secrets.

Historical methods

The notion of a transcendental number has been slow to construct. This concept reflects the discoveries of confusing numbers, which have escaped the control and the understanding of mathematicians for centuries. The term *transcendante* was finally first introduced by G. Leibniz in 1682 to refer to certain curves, as graphs of sinus and cosinus, in contrast to algebraic curves. But it was not until 1844 that J. Liouville gave a formal definition of these puzzling numbers. These are the complex numbers which are not roots of any polynomial with integers as coefficients. The complement of this set is the one of algebraic numbers over $\mathbb{Q}$, denoted as $\overline{\mathbb{Q}}$.

From J. Liouville to K. Weierstrass

Among the first examples of transcendental numbers provided by Liouville, there is $\sum_{k=1}^{+\infty} 2^{-k!}$. The proof relies on a fundamental theorem of diophantine approximation called the Liouville inequality. Let $\alpha \in \overline{\mathbb{Q}}$, and let n be its degree, that is the minimal degree among the ones of the non-zero polynomials $P(X) \in \mathbb{Q}[X]$ such that $P(\alpha) = 0$. Then, the inequality of Liouville states that there exists a constant $c > 0$ such that for all $p \in \mathbb{Z}, q \in \mathbb{N}^*$ with $p/q \neq \alpha$, we have

$$\left|\alpha - \frac{p}{q}\right| > \frac{c}{q^n}. \tag{1}$$

Indeed, the decrease of $2^{-k!}$ makes the series *too much well approached* by rational numbers to be algebraic over $\mathbb{Q}$.

Note that the Liouville inequality was improved in 1955 by K. Roth who replaced n by $2 + \epsilon$ in (1), for any $\epsilon > 0$. Furthermore, it was generalised in 1972 as a simultaneous approximation result to several algebraic numbers by rational numbers with the same denominator by W. Schmidt and his subspace theorem.

In 1874, G. Cantor established that the set of real numbers is not countable, whereas the set $\overline{\mathbb{Q}}$ of algebraic numbers is. Hence, he showed that almost all real or complex numbers are transcendental.

Padé approximants

In Hermite's proof, Padé approximants refer to the simultaneous approximation to integral values of the exponential function by rational fractions with the same denominator. In the proof of the Lindemann–Weierstrass theorem, they arise from functional linear approximating forms for $e^{\rho_1 z}, \ldots, e^{\rho_n z}$.

Later, Padé approximants are no longer explicitly constructed but seen as solutions of linear systems with fewer equations than unknowns. This is the famous Pigeonhole principle. This was completed by C. Siegel, who gave an upper bound for the coefficients of such a solution, depending on the system. In particular, this allowed Siegel to prove in 1929 that the set of algebraic integral points on an algebraic curve of genus $g > 0$ over $\mathbb{Z}$ is finite. Note that the second crucial argument of his proof is a refinement of the Liouville inequality.

However, one of the first big events of the theory of transcendence had already taken place one year earlier with the proof of the transcendence over $\mathbb{Q}$ of the number e by C. Hermite. The essential ingredient, nowadays called *Padé approximants* (see

[1] *Flore et Pomone*, Colette. Translation: *What is the majesty of what ends, compared to the staggering departures, to the disorders of the dawn?*

the aside), is the key of the evolution of the theory of transcendence. Following the same method, F. Lindemann proved in 1882 that π is transcendental over $\mathbb{Q}$. Then, he established the transcendence of the values of the exponential function at non-zero algebraic points. Lindemann also stated a more general result, but without a complete proof, which was written later by Weierstrass.

This result, called the Lindemann–Weierstrass theorem, states that, given linearly independent algebraic numbers $\rho_1, \ldots, \rho_n$ over $\mathbb{Q}$, the numbers $e^{\rho_1}, \ldots, e^{\rho_n}$ are algebraically independent over $\overline{\mathbb{Q}}$. This means that there is no non-zero polynomial $P(X_1, \ldots, X_n) \in \overline{\mathbb{Q}}[X_1, \ldots, X_n]$ such that $P(e^{\rho_1}, \ldots, e^{\rho_n}) = 0$.

The Lindemann–Weierstrass theorem provides an example of algebraically independent functions that take algebraically independent values at non-zero algebraic numbers. This transfer of algebraic independence from functions to their values at some algebraic points is not true for arbitrary functions. However, there exist at least three cases in which this holds: certain differential systems (E-functions), Mahler systems and τ-difference systems (in positive characteristic). In what follows, we will present these three settings, starting with the ones which appear in characteristic zero. Our aim is to stress the major developments they bring to the theory of transcendence and algebraic independence.

C. Siegel's work and A. Shidlovskii's contribution

In the same paper of 1929, Siegel introduces a new class of functions, called E-functions, that are, in particular, solutions of linear differential systems with coefficients in $\overline{\mathbb{Q}}(z)$. Examples of E-functions are the exponential and the Bessel functions, along with some hypergeometric functions.

The work of Siegel was completed later by Shidlovskii [4] and gives the following fundamental result, known as the Siegel–Shidlovskii theorem. Let $(f_1(z), \ldots, f_n(z))$ be a vector of E-functions, which satisfies a linear differential system. Let α be a non-zero algebraic number that is not a pole of the matrix of the system. Then the transcendence degree of $\{f_1(\alpha), \ldots, f_n(\alpha)\}$ over $\overline{\mathbb{Q}}$ is equal to the one of $\{f_1(z), \ldots, f_n(z)\}$ over $\overline{\mathbb{Q}}(z)$. Recall that the transcendence degree of a family is the maximal number of algebraically independent elements it contains. Note that Y. André recovered this statement by studying the singularities of the concerned differential system.

K. Mahler's method and Ku. Nishioka's theorem

At the same time, Mahler introduced a method similar to the one of Siegel, which applies to (multivariate) functions, solutions of difference equations, rather than differential ones, of the form $f(\Omega z) = \frac{\sum_{l=0}^{m} a_l(z) f(z)^l}{\sum_{l=0}^{m} b_l(z) f(z)^l}$, where $z = (z_1, \ldots, z_n)$, $a_l(z), b_l(z)$ are polynomials with coefficients in a finite field extension of $\mathbb{Q}$, and $\Omega = (r_{i,j}) \in \mathcal{M}_n(\mathbb{N})$ acts over $\mathbb{C}^n$ as $\Omega z = \left(\prod_{j=1}^{n} z_j^{r_{1,j}}, \ldots, \prod_{j=1}^{n} z_j^{r_{n,j}}\right)$. Moreover, Mahler requires that the biggest eigenvalue ρ associated with Ω is real and that $1 \leq m < \rho$.

Mahler's first main result guarantees, under some conditions, the transcendence of the values of a transcendental Mahler function at algebraic points. However, contrary to E-functions, values of Mahler functions are not specially important numbers for applications. For example, a priori, we cannot obtain, π as a value of such functions at an algebraic point. Yet, Mahler had two stimulating goals for his method. The first one concerned functions of the type of the Jacobi Θ function, as for example $\Theta(z_1, z_2) = \sum_{n=0}^{+\infty} z_1^{2n} z_2^{n^2}$. Indeed, $\Theta(\Omega(z_1, z_2)) = \frac{\Theta(z_1, z_2) - 1}{z_1^2 z_2}$, where $\Omega = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. But the greatest eigenvalue of Ω is $\rho = 1$, contrary to the assumptions of Mahler.

The second target of Mahler was the Fourier expansion of the modular invariant J attached to an elliptic curve, of which he conjectured the transcendence at algebraic points of the unit disc, except zero. Indeed, this function for every $q \in \mathbb{N}^*$ satisfies an equation of the form $P_q(J(z), J(z^q)) = 0$, where $P_q(X, Y) \in \mathbb{Z}[X, Y]$. Mahler mentioned but never studied polynomial equations of this kind; on the other hand Nishioka [3] obtained transcendence results for values of solutions of such equations, under some assumptions. Unfortunately, these are not satisfied by P_q. Nevertheless, it is interesting to note that this conjecture of Mahler was proved in 1996 by K. Barré-Sirieix, G. Diaz, F. Gramain and G. Philibert using similar steps as the ones of Mahler's method. Thus, we can consider it as a fruitful source of inspiration.

Mahler's method finally came to prominence in 1969, when W. Schwarz proved transcendental results for values of some functions, without mentioning or exploiting the fact that they are Mahler functions. Then, Mahler published a paper in which he recalled his method and explained why the results of Schwarz

are in fact corollaries of his own work. Mahler also suggested three open problems to generalise his approach. These gave rise to several works, from K. Kubota and J. Loxton and A. van der Poorten, who extended some results of Mahler and contributed to the popularisation of his method. Further extensions were made later by M. Amou, P.-G. Becker, D. Masser, Ku. Nishioka, T. Töpfer, and many others. Through their impulse, the study moved to linear equations of the form

$$P_0(z)f(z)+P_1(z)f(z^d)+\cdots+P_m(z)f(z^{d^m})=0, \quad (2)$$

where $P_0(z),\ldots,P_m(z)\in\mathbb{K}[z]$, $P_m(z)\neq 0$ and $\mathbb{K}$ is a number field.

Moreover, this new enthusiasm around Mahler's method has been perpetuated by the links it shares with the dynamical research area of finite automata theory, especially highlighted by M. Mendès-France. Indeed, every generating series of an automatic sequence is a Mahler function in the sense of equation (2).

The apotheosis of this first period of research was reached by Nishioka, who proved in 1990 the analogue of the Siegel–Shidlovskii theorem for solutions of Mahler systems. Thereafter, these two results have been refined into statements saying that, under the same respective assumptions, every homogeneous algebraic relation over $\overline{\mathbb{K}}$ between the numbers $f_1(\alpha),\ldots,f_n(\alpha)$ arises from the specialisation at α of a homogeneous algebraic relation over $\overline{\mathbb{K}}\{z\}$ between the functions $f_1(z),\ldots,f_n(z)$, solutions of the differential or Mahler system concerned. For E-functions this refinement is due to F. Beukers in 2006 (based on the results of André). For Mahler functions it is obtained in 2017 by B. Adamczewski and C. Faverjon as a consequence of the work of P. Philippon. Recently André has recovered this result by developing a new Galois theory based on affine quasi-homogeneous varieties. Then, L. Nagy and T. Szamuely also have recovered it from a generalisation of this theory of André.

Characteristic p

A big part of the classical theory of transcendence and algebraic independence deals with periods in the sense of M. Kontsevich and D. Zagier. Examples of these remarkable numbers are π, values of logarithm at algebraic points, values of the Riemann zeta function at integral points, or powers of values of the Euler gamma function at rational points. In characteristic p, there exists a framework in which we can build a parallel of this arithmetical context and define analogues of the complex exponential and logarithm functions, the Riemann zeta function or the Euler gamma function, along with associated values such as π. This is the function fields setting which shares profound analogies with the number fields one. Let us illustrate this fact by Figure 1.

On the left, finite extensions $\mathbb{K}$ of K are number fields. On the right, they are called function fields (of one variable). Periods are there replaced by remarkable numbers called periods of Drinfeld modules. They are any elements of the free A-lattice $\Lambda\subseteq C$ of finite rank formed by the zeros of an exponential function defined in this setting.

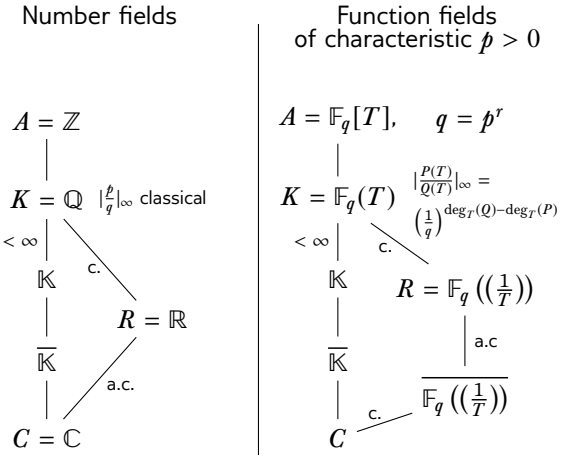

Figure 1. c. signifies completion for $|.|_\infty$, a.c. signifies algebraic closure.

As an illustration, the analogue of the complex exponential function is called the Carlitz exponential. The associated lattice is $\Lambda=\varpi A$, with $\varpi=T(-T)^{1/(q-1)}\prod_{i=1}^{+\infty}\left(1-T^{1-q^i}\right)^{-1}$. Consequently, the period ϖ is an analogue of $2i\pi$ and $\pi_q:=\prod_{i=1}^{+\infty}\left(1-T^{1-q^i}\right)^{-1}$ is an analogue of π.

As the second illustration one can consider Drinfeld modules whose associated lattice has rank 2. Indeed, they are analogues of elliptic curves, and periods, quasi-periods and a j-invariant can be attached to them. In 1998, M. Ably, L. Denis and F. Recher, obtained the transcendence of values of the Taylor expansion J associated with j for algebraic numbers α such that $0<|\alpha|<(1/q)^{1/(q-1)}$. This is the positive characteristic analogue of the result due to Barré-Sirieix, Diaz, Gramain and Philibert in characteristic zero, mentioned previously.

Beyond these similarities, there exist difficulties that specifically appear in the setting of function fields of positive characteristic. A significant example of such as difficulty is the fact that the statement of Roth and consequently the one of Schmidt's subspace theorem is no longer satisfied. In spite of that, algebraic independence results in the function fields setting are often more developed than their analogues in the number fields framework, which are still conjectures.

Let us now describe the methods developed in characteristic p to obtain results of transcendence and algebraic independence of remarkable numbers in this setting. We will compare these statements to their analogues in characteristic zero.

First results of L. Wade's and J. Geijsel's contributions

The first main results dealing with transcendence of periods of Drinfeld modules are due to L. Wade in the 1940s. To begin with, Wade proved that the values at non-zero algebraic points of the Carlitz exponential and logarithm were transcendental over $\overline{\mathbb{F}_q(T)}$. We recognize in the first statement the analogue of the classical Hermite's theorem. Wade also showed that the period ξ he associated with the Carlitz exponential was transcendental. Then, J. Geijsel stated that the set of the periods of the Carlitz exponential is $\mathbb{F}_q[T]\xi$. This extends the result of transcendence of Wade to all periods of the Carlitz exponential e_C. An other key result of this time was the analogue of the Gelfond–Schneider theorem, obtained by Wade, and generalised by Geijsel. It states that, given a non-zero element $\alpha \in C$ such that $e_C(\alpha) \in \overline{\mathbb{F}_q(T)}$ and $\beta \in \overline{\mathbb{F}_q(T)} \setminus \mathbb{F}_q(T)$, the number $e_C(\alpha\beta)$ is transcendental over $\overline{\mathbb{F}_q(T)}$.

The automatic method

Around 1990, J.-P. Allouche developed a new method to prove the transcendence of some numbers of the type $\alpha = \sum_{n=0}^{+\infty} a_n(1/T)^n \in \mathbb{F}_q((1/T))$ is based on two results. First one by G. Christol, which states that α is algebraic over $\overline{\mathbb{F}_q(T)}$ if and only if $(a_n)_n$ is q-automatic. The other result is one of S. Eilenberg which gives the following useful characterisation of automaticity. The sequence $(a_n)_n$ is k-automatic if and only if its k-kernel $K_k(\underline{a})$ is finite, where $K_k(\underline{a}) = \{(a_{k^r n+j})_n, r \geq 0, 0 \leq j < k^r\}$.

In particular, this strategy allowed Allouche to recover the transcendence of π_q first obtained by Wade. This approach was then generalised by V. Berthé, who recovered the transcendence over $\overline{\mathbb{F}_q(T)}$ of quotients $\zeta_C(s)/\pi_q^s$ for $s \in \{1, \dots, q-2\}$, and the one of the elements $\log_C(P)/\pi_q^s$, for $s \in \{1, \dots, q-2\}$, $q > 3$ and certain rational fractions $P \in \mathbb{F}_q(T)$.

The method of G. Anderson, W. D. Brownawell and M. Papanikolas

A very powerful and fruitful method in the framework of function fields of positive characteristic has been developed by Anderson, Brownawell and Papanikolas [1]. It deals with τ-difference systems, that is

$$\begin{pmatrix} \tau f_1(z) \\ \vdots \\ \tau f_n(z) \end{pmatrix} = A(z) \begin{pmatrix} f_1(z) \\ \vdots \\ f_n(z) \end{pmatrix}, \tag{3}$$

where each $f_i(z)$ admits a series expansion around zero with coefficients in a function field of characteristic $p > 0$ and where for $f(z) = \sum_{n=0}^{+\infty} a_n z^n$, with $a_n \in C$, we set $\tau f(z) = \sum_{n=0}^{+\infty} a_n^{1/q} z^n$.

This method is based on the t-motives theory introduced by Anderson around 1986, which generalises the notion of Drinfeld modules. This gave rise to analogues of classical results. For example, in 1997, J. Yu used this method to establish the analogue of the well-known A. Baker's theorem. Besides, the three previous authors prove in 2004 the analogue of the refinements of the Siegel–Shidlovskii and Nishioka's theorems for systems (3). In addition, the approach of these authors provides results which widely overcome the classical results of algebraic independence. Let us describe some of them. To begin with, L. Denis manipulated this theory to state in 1995 that $e = e_C(1)$ and ϖ are algebraically independent over $\overline{\mathbb{F}_q(T)}$. Moreover, in 2007, C.-Y. Chang and J. Yu considered t-motives to give a complete description of all the algebraic relations over $\overline{\mathbb{F}_q(T)}$ between values at integers of the analogue in this setting of the Riemann function, called the Carlitz zeta function.

Furthermore, Papanikolas used this method in 2008 to complete the analogue of Baker's theorem. More precisely, he stated that, if $\alpha_1, \dots, \alpha_n \in \overline{\mathbb{F}_q(T)}$ are in the disc of convergence of the Carlitz logarithm $\log_C$ and if $\log_C(\alpha_1), \dots, \log_C(\alpha_n)$ are linearly independent over $\mathbb{F}_q(T)$, then these logarithms are algebraically independent over $\overline{\mathbb{F}_q(T)}$ at algebraic points.

A lot has also been found concerning algebraic independence of periods ω_1, ω_2 and quasi-periods

η_1, η_2 attached to Drinfeld modules ϕ with lattice Λ of rank 2. The main result was obtained by Chang and Papanikolas (see the aside) in 2012, when they proved that ω_1, ω_2, η_1, η_2 are algebraically independent, in the case where the characteristic p is odd and ϕ is without complex multiplication. This means that we have $\{c \in C, c\Lambda \subseteq \Lambda\} = \mathbb{F}_q[T]$. The complex multiplication case, that is, $\mathbb{F}_q[T] \subsetneq \{c \in C, c\Lambda \subseteq \Lambda\}$, is different. Indeed, A. Thiery proved in 1992 that then the transcendence degree over $\overline{\mathbb{F}_q(T)}$ of the family $\{\omega_1, \omega_2, \eta_1, \eta_2\}$ equals 2.

The result of Chang and Papanikolas

In addition to the use of t-motives, their proof relies on the fundamental theorem of Anderson, Brownawell and Papanikolas mentioned previously (which traces every homogeneous algebraic relation among values of solutions of τ-difference systems back to a functional relation), and a strong result of Papanikolas, which turns the study of algebraic relations among periods and quasi-periods of Drinfeld modules ϕ into the study of a difference Galois group attached to a τ-difference system associated to ϕ.

By comparison, in the classical case, we are very far from such results. Indeed, the algebraic independence over $\overline{\mathbb{Q}}$ of e and π or of values of logarithms are conjectured but not proved. Concerning values of the Riemann zeta function, the only general result is the transcendence of its values at even integers. R. Apéry established in 1978 that $\zeta(3)$ is irrational. It 2001, it was proved by K. Ball and T. Rivoal that an infinite number of values of ζ at odd integers is irrational. However, we still do not know how to find other irrational values, apart from $\zeta(3)$. The closest result from this goal is due to W. Zudilin in 2004 and guarantees that at least one of the four numbers $\zeta(5)$, $\zeta(7)$, $\zeta(9)$, $\zeta(11)$ is irrational.

In the same vein, our knowledge concerning elliptic curves is limited. Let us first mention Schneider who proved in 1957 the transcendence of every period associated to an elliptic curve. Another significant result was established by G. Chudnovsky in 1984 and gives the algebraic independence of $\frac{\pi}{\omega}$ and $\frac{\eta}{\omega}$ for every pair (ω, η) of period and quasi-period of an elliptic curve, when some invariants attached to its lattice are algebraic. Under this latter assumption, this implies the existence of at least two algebraically independent numbers among generator periods and associated quasi-periods. However it seems that no more general results appear in this setting.

Mahler's method in positive characteristic

One of the advantages of Mahler's method rests in the fact that its validity does not depend on the characteristic involved. Moreover, a fruitful observation by Denis strongly motivates the use of this method in positive characteristic. It is the fact that, in the setting of function fields of positive characteristic, some periods such as π_q, arise from specialisations of Mahler functions at algebraic points. This makes Mahler functions quite natural in this framework, more than in the classical case. Through the innovative and prolific directions of the work of Denis, Mahler's method has contributed since the 1990s to the first statements of algebraic independence that overcome the classical case.

However, beyond the analogies between number fields and function fields of positive characteristic, some established results in the second setting do not admit any translation in the first one, because they concern objects that have no classical equivalents. For example, let us notice that the analogue $\pi_q \in \mathbb{F}_q((1/T))$ of π depends on q. Thus, by changing q, we obtain several analogues of π. Then, Denis shows that, if $i_1, \ldots, i_n$ is a strictly increasing sequence of non-zero natural numbers, then $\pi_{q^{i_1}}, \ldots, \pi_{q^{i_n}}$ are algebraically independent over $\overline{\mathbb{F}_q(T)}$. Finally, using Mahler's method, G. Fernandes [2] recently established the analogues of Nishioka's theorem and its refinement in the function fields setting.

Related topics and perspectives

Algebraic independence measures

Let us consider either one of the two settings introduced in Figure 1. An algebraic independence measure of numbers $\xi_1, \ldots, \xi_n \in C$ is a lower bound of the form $|P(\xi_1, \ldots, \xi_n)| \geq \phi(\deg(P), H(P))$, where ϕ is a real function, which is satisfied for every polynomial $P \in \overline{\mathbb{K}}[X_1, \ldots, X_n]$ such that $P(\xi_1, \ldots, \xi_n) \neq 0$. Recall that $\deg(P)$ is the total degree of P and $H(P)$ is the maximal of the absolute values of the coefficients of P.

Let us restrict ourselves to values of E-functions, Mahler functions, and solutions of τ-difference systems in positive characteristic. As far as we know,

the third case did not generate general results on this topic.

In the first two cases, in characteristic zero, such algebraic independence measures are due to Shidlovskii (using Siegel's method), and Nishioka (using Mahler's method) respectively, for algebraically independent functions. E. Zorin is now working on a general method, which, under some conditions, would in particular generalize the measures of Nishioka to algebraically dependent Mahler functions, in characteristic zero. Finally, in positive characteristic, only partial results are available. For example, P.-G. Becker obtained in 1994 algebraic independence measures for Mahler functions with coefficients in $\mathbb{F}_q$. Furthermore, Denis used Mahler's method in an interesting way in 2011 to get algebraic independence measures for some inhomogeneous Mahler equations of order 1. A promising direction of study could be to adapt the works of Zorin and Denis to obtain general algebraic independence measures for values of Mahler functions in positive characteristic.

Galois theory

If we focus on methods developed by Siegel, Nishioka and Anderson, Brownawell and Papanikolas, we note that the results and refinements obtained turn the problem of the algebraic independence of numbers into the one of the algebraic independence of functions. This latter question is at the heart of the Galois theory of functional systems. Indeed, its main idea is to attach to the concerned system a linear algebraic group that reflects the algebraic relations between the solutions. This group is called the Galois group associated to the system.

Some authors apply the general ideas of this theory in the particular cases of E-functions, Mahler functions, or solutions to τ-difference systems in positive characteristic. Concerning the first setting, the Hrushovski algorithm developed in 2002 computes the algebraic relations between solutions of differential systems. Very recently, S. Fischler and T. Rivoal adapted it (starting from the modified version by R. Feng) and deduced from Beuker's theorem an algorithm which computes a set of generators of the algebraic relations over $\overline{\mathbb{Q}}$ between values of E-functions at algebraic points. For Mahler functions in characteristic zero, the work of J. Roques proves fundamental properties of the associated Galois group. Roques also studied in detail the case of Mahler equations of order 2. Finally, for τ-difference systems in characteristic p, as mentioned previously, a Galois theory is developed by Papanikolas. In addition, let us mention the Tannakian Galois theory, treated in particular by C. Hardouin. This approach is very general and especially includes the differential and τ-difference cases. However, it seems that no Galois theory has been developed in the precise case of Mahler functions in positive characteristic. This could be a further direction of study.

To conclude, we should stress that, even if Galois theory is a powerful tool to deal with algebraic independence of functions that satisfy some functional equations, it is in general difficult to compute the associated Galois group. That is why, so far, explicit results of functional algebraic independence do not overcome equations of order 2, for the three types of equations mentioned previously. The way is then widely open to fabulous discoveries!

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Markov Bases of Toric Ideals: connecting commutative Algebra and Statistics

DIMITRA KOSTA

Algebraic statistics is a rapidly evolving new field which uses tools from algebraic geometry and commutative algebra to address problems in statistics, probability and their applications. Markov bases constitute one of the first connections between commutative algebra and statistics. In the current article, we focus on Markov bases of monomial curves in affine space, and on a notion called Markov complexity.

Contingency tables

In statistics, a contingency table is a table in a matrix format that is used to record and analyse the relationship between two or more categorical variables. As an example, suppose that 100 individuals from a large population are randomly sampled to discover whether there is any correlation between handedness and gender at birth. One would record the results of this sampling in a contingency table, like the one displayed below.

	Left-handed	Right-handed	Total
Male	9	43	52
Female	4	44	48
Total	13	87	100

Suppose now we wish to answer the question of whether there is any relationship between gender at birth and handedness. Let u_{ij}, $i = 1, 2$, $j = 1, 2$ be the frequencies of the four cells in a 2×2 contingency table. The row and the column sums are denoted by $u_{i+}, i = 1, 2$ and $u_{+j}, j = 1, 2$ respectively and are called marginal frequencies. Suppose our statistical model gives the expected cell frequencies e_{ij}. In statistics, a χ^2- test evaluates how likely it is for two variables to be independent of each other. The formula for the Pearson's χ^2-statistic for our contingency table is

$$\chi^2 = \sum_{i,j} \frac{(u_{ij} - e_{ij})^2}{e_{ij}}.$$

To answer the question of whether gender and handedness are related, one approach is to let the sample size tend to infinity. Then, under the independence assumption, Pearson's χ^2-statistic converges to the χ^2-distribution which does not depend on the true underlying distribution. This allows one to reject or accept the independence hypothesis with some confidence level. Although theoretically this makes sense, data can be rare or expensive to obtain, so letting the sample space tend to infinity is not realistic.

An alternative approach when the sample size is small is to use Fisher's exact test. In this case, in order to decide how likely it is to accept or reject the independence hypothesis, one needs to take a sum over the space of all contingency tables with given marginal frequencies. This space is called the *fibre* of the given table u and is denoted by $\mathcal{F}(u)$. However, the fibre can be enormous, in which case it is impossible to write down its elements. The usual approach then is to generate random samples from the fibre $\mathcal{F}(u)$ with respect to the distribution dictated by the statistic, in order to obtain a Monte Carlo estimate of the p-value (this is a value which is used to make a decision on whether to accept or reject the independence hypothesis). A key step in this approach is the generation of random samples of contingency tables with fixed marginal frequencies, which is a non-trivial task. To this end, new tools have been developed based on techniques from commutative algebra and computational algebraic geometry.

Markov bases

The seminal paper by Diaconis and Sturmfels proposes an algebraic algorithm for constructing a connected Markov chain in order to produce random samples from the fibre $\mathcal{F}(u)$. This algorithm relies on the discovery of a Markov basis which is a set of moves connecting any two contingency tables which have the same column sums and row sums. Markov bases are connected to commutative algebra through algebro-geometrical objects called toric ideals. In particular, the fundamental theorem of

Markov bases established in [4] states that a Markov basis corresponds to a system of generators of a toric ideal associated to an integer matrix. The importance of this correspondence is that it guarantees that a Markov basis exists due to the Hilbert basis theorem in algebraic geometry. So, there is always a finite set of moves that connects all of the fibres $\mathcal{F}(u)$.

Let $A = [a_1, \ldots, a_n] \in \mathbb{N}^{m\times n}$. Write any element $u \in \mathrm{Ker}_{\mathbb{Z}}(A)$ in the kernel of A as $u = u^+ - u^-$, where $u^+, u^- \in \mathbb{N}^n$. The toric ideal corresponding to A is the ideal in $k[x_1, \ldots, x_n]$, with k a field, defined by

$$I_A = \langle x^{u^+} - x^{u^-} | u \in \mathrm{Ker}_{\mathbb{Z}}(A)\rangle.$$

A *Markov basis* of A is a finite subset M of $\mathrm{Ker}_{\mathbb{Z}}(A)$ such that whenever $w, u \in \mathbb{N}^n$ and $w - u \in \mathrm{Ker}_{\mathbb{Z}}(A)$, there exists a subset $\{v_i : i = 1, \ldots, s\}$ of vectors from M that connects w to u. This means that $(w - \sum_{i=1}^{p} v_i) \in \mathbb{N}^n$ for all $1 \le p \le s$ and $w - u = \sum_{i=1}^{s} v_i$.

We call a Markov basis M of A *minimal* if no subset of M is a Markov basis of A. The union of all minimal Markov bases of A is called the *universal Markov basis* of A and is denoted by $M(A)$.

Example of a Markov basis

For $A = [3, 4, 5]$ we have that the kernel is $\mathrm{Ker}_{\mathbb{Z}}(A) = \langle(1, -2, 1), (3, -1, -1)\rangle$ and the corresponding toric ideal is

$$I_A = \langle x_1x_3 - x_2^2, x_1^3 - x_2x_3, x_1^2x_2 - x_3^2\rangle.$$

A minimal Markov basis for I_A is $\{(-3, 1, 1), (1, -2, 1), (2, 1, -2)\}$.

Lawrence liftings

In statistics, one is interested in studying the Markov bases of some configurations called higher Lawrence liftings. The *r-th Lawrence lifting* of A is denoted by $A^{(r)}$ for $r \ge 2$ and is the $(r + n) \times rn$ matrix below.

$$A^{(r)} = \begin{pmatrix} \overbrace{\begin{matrix} A & 0 & & 0 \\ 0 & A & & 0 \\ & & \ddots & \\ 0 & 0 & & A \end{matrix}}^{r-\text{times}} \\ I_n \quad I_n \quad \cdots \quad I_n \end{pmatrix},$$

For instance, consider the independence model for two discrete random variables with 2 states. For the marginal frequencies of the 2×2 contingency table $u = (u_{ij})_{i,j=1,2}$, we have

$$\begin{pmatrix} u_{1+} \\ u_{2+} \\ u_{+1} \\ u_{+2} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_{11} \\ u_{12} \\ u_{21} \\ u_{22} \end{pmatrix}.$$

The 0/1-matrix above is the 2-nd Lawrence lifting $[1, 1]^{(2)}$. The Markov basis of $[1, 1]^{(2)}$ is $\{1, -, 1, -1, 1\}$, giving random moves on the space of 2×2 contingency tables that do not change the marginal distribution. Each single move is given by adding or subtracting the matrix $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ with probability $1/2$ each.

Similarly for $I \times J \times r$ contingency tables we are interested in the Markov basis of the r-th Lawrence lifting of the configuration A corresponding to the $I \times J$ two-way independence model [1, Section 9.8].

We identify an element of $\mathrm{Ker}_{\mathbb{Z}}(A^{(r)})$ with an $r \times n$ matrix. Each row of this matrix corresponds to an element of $\mathrm{Ker}_{\mathbb{Z}}(A)$ and the sum of its rows is zero. The *type* of an element of $\mathrm{Ker}_{\mathbb{Z}}(A^{(r)})$ is the number of non-zero rows of this matrix.

4-th Lawrence lifting of $A = [3, 4, 5]$

$$\begin{pmatrix} 3 & 4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 5 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

We identify the element of $\mathrm{Ker}_{\mathbb{Z}}([3, 4, 5]^{(4)})$

$$\begin{pmatrix} 1 & -2 & 1 & 2 & 1 & -2 & 0 & 0 & 0 & -3 & 1 & 1 \end{pmatrix}^T$$

with the 4×3 matrix:

$$\begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 0 & 0 & 0 \\ -3 & 1 & 1 \end{pmatrix}$$

It has three non-zero rows, so it has type 3.

Markov complexity

In the first section we considered a two-way interaction model as a guiding example; we had two categorical variables each having two states. An advantage of the algebraic algorithm by Diaconis and Sturmfels is that it appeared as the only way to sample from three- and higher way contingency tables. Consider now the model of no-three-factor interaction and the case of a three-way contingency table $I \times J \times K$ where $I, J, K \in \mathbb{N}$ are the states of the three categorical variables. Aoki and Takemura studied the structure of a minimal Markov basis for this model for $3 \times 3 \times K$ tables for $K \geq 3$ (see [1, Chapter 9]). They showed that the structure of a minimal Markov basis for $3 \times 3 \times K$, $K > 5$ contingency tables, is essentially explained by $3 \times 3 \times 5$ tables. However, we do not yet have a closed-form expression for a Markov basis for the no-three-factor model for general $I \times J \times K$ tables.

The result by Aoki and Takemura gave rise to the notion of a Markov complexity and highlighted the need to develop the theory of Markov bases and their complexity.

Definition 1. The *Markov complexity* $m(A)$ of A is the largest type of any vector in the universal Markov basis of the r-th Lawrence lifting $A^{(r)}$ as r varies.

In our notation, the result by Aoki and Takemura states that for the 3-rd Lawrence lifting $A = [1,1,1]^{(3)}$, the type of any element in a Markov basis of $A^{(r)}$ as r varies is at most 5; namely, the Markov complexity $m(A)$ of A is 5. This is the first instance of a matrix with finite Markov complexity. In 2003, Santos and Sturmfels [7] showed that the Markov complexity of any matrix A is finite, which is an extraordinary result given that A can be any matrix configuration in $\mathbb{N}^{m \times n}$ and we take as high a Lawrence lifting as we want. However, computing the complexity $m(A)$ of Markov bases is a challenging problem; no formula for $m(A)$ is known in general and there are very few classes of toric ideals in the literature for which the complexity has been computed [1, 2, 7].

A geometric characterisation for the Markov complexity of monomial curves in $\mathbb{A}^3$ is given in [2]. It is shown that the Markov complexity of any monomial curve $\mathcal{A} \in \mathbb{A}^3$ is equal to two if the toric ideal $I_{\mathcal{A}}$ is complete intersection, and equal to three otherwise, answering a question posed by Santos and Sturmfels [7, Example 6]. A complete intersection toric ideal I is a toric ideal generated by exactly codim$V(I)$ polynomials, where $V(I)$ is the variety defined by the ideal I.

In order to develop the theory of Markov bases the natural next step is to study the Markov complexity of curves lying in higher dimensional affine space. We start with (complete intersection) monomial curves in $\mathbb{A}^4$ with an aim to find a formula for the Markov complexity which ideally would depend on the codimension of the toric ideal.

Markov complexity of monomial curves

For $\mathcal{A} = [n_1, n_2, \ldots, n_d]$ a $1 \times d$ matrix of positive integers, we call the toric ideal $I_{\mathcal{A}}$ the toric ideal of a monomial curve. This is because the set of zeroes $V(I_{\mathcal{A}})$ in the d-dimensional affine space over an algebraically closed field k is the set $\{(t^{n_1}, \ldots, t^{n_d}) | t \in k\}$.

In my joint article with A. Thoma [6], we study the Markov complexity $m(\mathcal{A})$ of monomial curves in $\mathbb{A}^m$, $m \geq 4$ and demonstrate that the result of [2], which bounds the Markov complexity of complete intersection monomial curves in $\mathbb{A}^3$ by their codimension, is a special property of monomial curves in $\mathbb{A}^3$ and cannot be generalised to higher dimensions. In particular, we obtain that complete intersection monomial curves in $\mathbb{A}^4$ may have arbitrarily large Markov complexity. In fact, this leads to the following theorem which is the main result of our paper:

Theorem 1. *Monomial (complete intersection) curves in $\mathbb{A}^m, m \geq 4$, may have arbitrarily large Markov complexity.*

The main idea of the proof is to start with $m = 4$ and find a family $\mathcal{A}_r = [a_1(r), a_2(r), a_3(r), a_4(r)]$ of monomial curves in $\mathbb{A}^4$, where the numbers $a_1(r), a_2(r), a_3(r), a_4(r)$ depend on a parameter r, such that the Markov complexity of $\mathcal{A}_n$, the n^{th} member of the family, is at least n. After several months working with the computational commutative algebra package `4ti2`, we found one such family of curves,

$$\mathcal{A}_n = [1, n, n^2 - n, n^2 - 1].$$

Through our `4ti2` computations, we were able to find an element of type n in $\mathrm{Ker}_{\mathbb{Z}}(\mathcal{A}_n^{(n)})$. Then, using the theory of Markov bases, we proved that this type n element belongs to every Markov basis of $\mathcal{A}_n^{(n)}$.

In particular, for the complete intersection monomial curve $\mathcal{A}_n = [1, n, n^2 - n, n^2 - 1]$, we showed that the type n element in $\mathrm{Ker}_{\mathbb{Z}}(\mathcal{A}_n^{(n)})$,

$$u = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ & \ddots & & \\ 1 & -1 & -1 & 1 \\ 0 & 0 & n+1 & -n \\ 2-n & n-2 & -3 & 2 \end{pmatrix},$$

cannot be split into a semi-conformal decomposition. The notion of a semiconformal decomposition gives an algebraic characterisation of the elements that belong to every minimal Markov basis. These elements correspond to moves in the Markov chain which are indispensable, namely they connect fibres with only two elements.

In more detail, let $u, v, w \in \mathrm{Ker}_{\mathbb{Z}}(A)$. We say that $u = v +_{sc} w$ is a *semiconformal decomposition* of u if $u = v + w$ and $v(i) > 0$ implies that $w(i) \geq 0$ and $w(i) < 0$ implies that $v(i) \leq 0$ for $1 \leq i \leq n$. Here $v(i)$ denotes the i^{th} coordinate of v.

We showed that if the vector u can be split semi-conformally as $u = v +_{sc} w$, then either $v = 0$ or $w = 0$. This implies that the type n element u belongs to every minimal Markov basis of $\mathcal{A}_n^{(n)}$. Since u has type n, the Markov complexity $m(\mathcal{A}_n)$ is at least n.

It should be highlighted that we do not claim that the Markov complexity $m(\mathcal{A}_n)$ of $\mathcal{A}_n$ is n, but is at least n. Indeed, consider the example of the monomial curve $\mathcal{A}_5 = (1, 5, 20, 24) \in \mathbb{A}^4$. Then using the computational package `4ti2`, we compute a Markov basis of $\mathcal{A}_5^{(r)}$, for $r \leq 6$.

The following table includes the number of elements of the Markov basis of $\mathcal{A}_5^{(r)}$ as well as the largest type of any vector in the universal Markov basis of $\mathcal{A}_5^{(r)}$, for each $r = 1, \ldots, 6$.

r^{th} Lawrence lifting	# elements of Markov basis	Type
2	46	2
3	174	3
4	528	4
5	1520	5
6	4110	6
7	10206	6

Given that the Markov complexity $m(\mathcal{A}_5)$ of $\mathcal{A}_n$ is the largest type of any vector in the universal Markov basis of $\mathcal{A}_5^{(r)}$ as r varies, this implies that the Markov complexity of $\mathcal{A}_n$ is at least 6.

Results on Markov bases using an algebraic approach have recently been applied to a wide range of settings. For instance, the recent review article [3] presents applications to network problems through a connection of networks to contingency tables. Moreover, apart from applications in statistics, applications for Markov bases have recently been found in algebraic geometry (see [5] for more details).

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Dimitra Kosta

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DeMorgan@21: Twenty-One Years of Success

JOHN JOHNSTON

On 19 October the DeMorgan@21 event to celebrate the twenty-first anniversary of the Society moving to its current headquarters was held at De Morgan House. This wonderful occasion was attended by LMS Officers and staff (past and present), as well as members and other friends of the Society. Here the LMS Society Communications Officer describes the event.

Over seventy people attended and were treated to reminiscences about the move to De Morgan House, and how the Society has expanded over the past 21 years to now provide a wide range of services and activities to the mathematical community. This is in no small part due to the foresight of the LMS Officers at the time seizing the opportunity to move the Society to its current location.

The afternoon was hosted by current LMS Vice President Cathy Hobbs who introduced a range of speakers who played an important role in the Society's move to De Morgan House. Before the reminiscences June Barrow-Green started proceedings with a presentation, entitled 'The Wit and Wisdom of Augustus De Morgan'. This was a journey through the musings of the Society's first President on mathematics and a range of other subjects, including his correspondence with notable individuals of the day.

John Ball was the LMS President who oversaw the purchase of De Morgan House and he gave his recollections of Council discussions during this time, which were not always straightforward, and the process of visiting a number of properties before deciding to visit a property in Russell Square, which although over budget seemed the ideal location for the Society. And the rest is history. Susan Oakes was the Society's administrator for almost twenty years and was one of the main players in facilitating the move to De Morgan House, and she recalled just the sheer amount of space that was available compared with the small rooms at Burlington House, and that the potential to expand the number of staff and activities was considerable. After the purchase was complete Martin Taylor became LMS President in 1998 and managed the Society's move to De Morgan House. He too was impressed by the 'many spacious rooms' and the possibility of letting some of the upper floors of the building, and also the potential of hosting lectures and conferences, which has indeed provided a good source of income for the Society over the past 21 years. It occurred to him that one of these spacious rooms might be nice for a President's office. Unfortunately for Martin, none was allocated for this purpose.

The opening ceremony in 1998 was attended by UK Fields Medallists and the building was opened formally by Professor Sir Michael Atiyah. In respect of this, the most recent UK Fields Medallist Caucher Birkar gave a short presentation about his work and life and how he had benefited from the LMS Cecil King Fellowship earlier in his career.

In the final presentation, current LMS President Caroline Series outlined how successful the move to De Morgan House had been, and how the foresight of those involved has been more than justified. See the Afterword on page 41 for further details.

De Morgan House

After the formal part of the event the attendees enjoyed a reception where they could catch up with old friends and colleagues. LMS President-Designate Jon Keating toasted the Society's past successes and looked to a future with the Society continuing to be at the centre of the mathematics community.

De Morgan House Anniversary: A Celebration

EXCERPTS FROM THE COMMEMORATION BOOKLET

As part of the DeMorgan@21 celebration, a commemoration booklet was prepared by the organising committee: June Barrow-Green, John Johnston, Susan Oakes, Caroline Series and Sarah Zerbes. It contains excerpts from past newsletters and Council minutes, as well as reminiscences of many members who were involved at the time of the move. We share excerpts of it here.

Introduction

On 16 February 1998, the London Mathematical Society moved from its previous premises in Burlington House, rented from the Royal Astronomical Society, and purchased the long leasehold on 57–58 Russell Square in 1998, renaming the property De Morgan House in honour of the first LMS President Augustus De Morgan. The official opening was celebrated with a gathering on 23 October of that year.

It was decided to mark the 21st anniversary of this occasion, momentous in the life of the LMS, by a gathering to reminisce and to celebrate how far the LMS has come since then. Who would have imagined then the extent to which the LMS has expanded its role, which has so much depended on the foresight in purchasing the premises?

Initially housing only a handful of LMS staff, the six-level Grade II listed twin Georgian terraces were chosen for three main reasons: their proximity to the LMS Library at UCL; to allow room to expand the LMS administration; and to serve as a fitting and prestigious meeting space and headquarters for the Society. Some space on the upper floors was given to commercial tenancies and today we have a full house of five commercial tenants ranging from like-minded mathematical bodies to city investment companies. We are particularly pleased to be able to provide office space to our colleagues in the IMA, and also to those in African Institute for Mathematical Sciences (AIMS) Africa.

The largest project undertaken since occupying the building was the 2005 redevelopment of the lower ground floor to create a suite of conference facilities. The project was completed and the new facilities launched in 2006. Now in their thirteenth year of operation, DMH Conference Facilities are a well-established central London venue partnered with one of the capital's leading catering companies. We host events of all shapes and sizes from small boardroom meetings to larger training days, product launches and summer receptions in the garden. Of course one of the main purposes of the space is use by the Society and the facilities still host all LMS Council and Committee meetings and many more LMS events. In addition we offer support to similar mathematical bodies with significant discounts which means that we host many other mathematical meetings and related events.

Figure 1. Opening commemoration plaque

As memorialised on the plaque (Figure 1) in the entryway, almost all of the British Fields medallists were present at the 1998 opening. Sadly, not all of them are still with us; in particular we regret the death of Sir Michael Atiyah at the beginning of this year. We also remember those involved in the move who are unable to attend the anniversary event for reasons of ill-health and age.

History of Russell Square and De Morgan House

In 1545, the manor of Bloomsbury was purchased by the Earl of Southampton from the Crown. The estate

passed through marriage to William Russell, 5th Earl (and later 1st Duke) of Bedford in 1669. Francis Russell, the 5th Duke of Bedford decided to develop the estate and the building agreement for Russell Square was signed between him and James Burton in June 1800. It is Francis' statue which overlooks the square opposite De Morgan House, showing him as an agriculturist with a sheep beside him and his hand on a plough, surrounding by four figures representing the four seasons spring, summer, autumn and winter.

De Morgan House comprises two of the nine terrace houses (Nos. 57 and 58), which run along the south side of the square near the corner with Southampton Row. The terrace was built by James Burton about 1800–1803. James in turn commissioned Humphrey Repton to design the Bloomsbury Square and Russell Square gardens which were completed in 1806. In the mid-19th Century, the continuous cast-iron railings were added and around 1898 the 11th Duke Herbrand Arthur Russell commissioned the architect Sir Philip Edward Pilditch to add the terracotta dressings following the building of the Russell Hotel by Charles Fitzroy Doll. In 1969, all of the buildings along the terrace were listed Grade II by English Heritage. The estate is now managed by Bedford Estates, and the buildings have to be maintained to their specifications.

Censuses from 1851–1901 show that both buildings were originally used as houses for upper middle class families who also had servants. The basement would have been the work area for the servants as part of the 'downstairs' household. The rooms are now used as conference rooms for Society Committee meetings and for external clients. Each room is named after a distinguished mathematician from the Society's history and in the Sylvester room, one can still see the remains of a huge fireplace which may have been the range in the kitchen.

The decision to move

Excerpt from the LMS Newsletter February 1998 by Alun Morris, LMS Treasurer

At an informal Special Meeting of the Council of the London Mathematical Society held in April 1997 to consider the future of the Society it was agreed that the time had come for the Society to purchase a building which would be at least large enough to cater for the Society's foreseeable needs. It was estimated that the overall cost of purchasing a suitable property in London would be around £1.5m.

At the following Council meeting in May, a sub-committee of Council was set up to look for suitable premises. By the October Council meeting, the subcommittee reported that a suitable building had been found, namely 57–58 Russell Square, which they could recommend as fulfilling the Society's requirements. At a Special Meeting of the Council held on 18 December 1997, the Council unanimously agreed that the Society should purchase the 112 year lease on 57–58 Russell Square for the sum of £1.8m. Contracts were exchanged on 16 January 1998.

Figure 2. The opening of De Morgan House.
Left to right: Richard Borcherds, Klaus Roth, Dan Quillen, Timothy Gowers, Michael Atiyah, Simon Donaldson, Alan Baker.

The premises are a Grade II listed building which have been refurbished to a very high standard. A presentation suite added in 1990 would not only serve as a Council chamber but as a venue for small meetings. The building is twice the size of that originally envisaged, thus approximately half of it will be sublet. It is possible that a suitable tenant has been found already. Members will be informed about the date when the Society's office will move and, of course, will be very welcome to visit our new premises.

The opening of De Morgan House

Excerpt from the LMS Newsletter December 1998 by Ben Garling, Executive Secretary

About 120 members of the London Mathematical Society and guests were present on Friday 23 October 1998 for the opening of De Morgan

House, the new headquarters of the Society, by Sir Michael Atiyah, OM, former President and De Morgan Medallist, Fields Medallist and former President of The Royal Society. Sir Michael unveiled the plaque in the lobby of De Morgan House in the presence of other Fields Medallists, Alan Baker, who returned especially from Japan, Richard Borcherds and Timothy Gowers awarded Fields Medals this year, Simon Donaldson, Daniel Quillen, Klaus Roth, who came down from Inverness for the occasion, and John Thompson, at present in the United Sates, but present in spirit.

Members and guests watched the ceremony on closed circuit television, in the Council Room and in the Members Room. After the formal opening, a reception was held, at which those present were able to explore the building and visit an exhibition about the history of the Society. During the reception, David Larman presented the Society with reproductions of pictures of Augustus De Morgan, the first President, on behalf of the Mathematics Department of University College, London.

Afterword by Caroline Series (LMS President 2017–19)

By any measure, the purchase of De Morgan House has been a resounding success. The foresight of those involved has been more than justified. DMH has enabled the LMS to expand enormously, so that we now have 18 staff providing support to our publishing enterprises and the 20+ LMS committees, besides organising events all around the country and dealing with many other activities. The expansion of staff has released academics from what would have become an impossible burden. As foreseen, we rent out the top rooms: besides offering space to our colleagues in the IMA and the AIMS we have three commercial clients.

Travel to the LMS headquarters has been made easier by DMH's close proximity to Euston, King's Cross and St Pancras. We are also just a short walk from a number of venues where on occasion we hire a larger lecture room or other venue, moreover we are in a relatively quiet and pleasant corner of London with a variety of hotels and restaurants close by. The flexible suite of rooms in the basement, including the Hardy room, accommodates a large number of different sizes and styles of meeting and entering the LMS Building on any given day one encounters a veritable hive of activity — frequently three or four different meetings are going on at the same time.

The comfortable Verblunsky Members' Room houses the Hardy collection, containing over 300 volumes from G.H. Hardy's personal library, and the Philippa Fawcett collection, a historical collection of books by and about women mathematicians. Among innovations since the move are the addition of the kitchen area, allowing in-house catering for lunches and receptions and even the occasional dinner, the installation of a lift, and more recently the refurbishment of the Cartwright and Rayleigh rooms. The Hardy room enables us to host medium size gatherings such as the Cartwright lectures, LMS Reps Days and two recent joint meetings with the IMA.

When the rooms are not in use for LMS purposes, they are let out, to the extent that DMH is now a successful meeting and conference venue in central London. Among those who regularly make use of our facilities are the National Centre for Excellence in the Teaching of Mathematics, the Mathematical Association, Mathematics in Education & Industry and the Solent Maths Hub. Other organisations include the Open University, UK Sport, the London Taught Course Centre and St Giles' English language courses. In all there are getting on for 500 clients who have used the building at one time or another, and lettings and conference bookings bring in a substantial and increasing income.

We are in the fortunate position of being able to offer our venue and sometimes services to other organisations; for example, the Hardy room is the regular venue for Council of Mathematical Sciences meetings. The value is enhanced by the recently installed video conferencing equipment which allows people to join meetings remotely. Our international links are also expanding. During my time as President, we have hosted meetings of the IMU Executive Committee and the Abel Prize Committee.

We have come a long way from the days of Susan Oakes in her office at the top of the elegant 71 step staircase in Burlington House. The LMS is taking on, and being asked to take on, ever more activities and roles, none of which would be possible without either De Morgan House and the facilities it offers, or our highly professional and willing staff, for which of course our expanded premises was a prerequisite.

The LMS is a great institution. Long may it thrive.

Reminiscence by Martin Taylor (LMS President 1998–2000)

My first sight of the premises of De Morgan House (DMH) occurred when I accompanied John Ball, who was then the President of the LMS, together with a number of the LMS officers, on a tour of inspection of our new premises. I was there in my capacity of President elect and I was very conscious of the fact that I would have oversight of moving everyone in and, above all, getting everyone settled. At that time Susan Oakes, Sylvia Daly and Harvinder Lotay worked in a tiny box-room of an office in Burlington House, which was rented to the LMS by the Royal Astronomical Society. The contrast with DMH was therefore enormous — where there were so many spacious rooms; indeed, conversation quickly turned to the possibility of letting some of the upper reaches of the building. The tour of all the rooms took well over half an hour, and, from time to time, the thought would occur to me: this might be nice for a President's office.

Then on 16 February 1998 we moved into DMH. There was a great sense of hustle and bustle as rooms were chosen for their various purposes and new office furniture was installed. When I gently enquired where my office was, I was told there wasn't one, but instead I was to have a kind of 'hot desk' in one of the lesser members' rooms! I realised then that Presidents may have great influence, but they have no real power.

Our new HQ not only had great space but it also had a superb location. Both I and a number of officers travelled down from the north, and DMH was an easy ten minute walk from Euston, Kings Cross and St Pancras. There were also a good number of fine restaurants nearby. We hosted numerous receptions in DMH — moving to the garden when the weather was clement. We would then all adjourn for the evening meal — going to a restaurant which would be only a minute to two away. It all worked so smoothly!

One of the great successes of the new building was the Members' Common Room: it had comfortable leather seats, old books lined the shelves, and the room had a cosy 'clubby' feel. Then I learned that, when we were to receive official guests, I was to use that room. That cheered me up quite bit. Two such occurrences that are still very fresh in my mind were: firstly, the time I had to receive Lord May, then the President of the Royal Society, and he commented on what a lovely room this was and he felt the Royal Society could do with something just like it! The second occasion occurred after the Guardian had used some bad statistical practice in assembling its league tables. My unenviable task, assisted by Harvey Goldstein, was to tick off the relevant Editor and his Assistant. Again, probably to ease the conversation, they commented on how impressed they were by the surroundings.

As we all settled into the routines of daily life in DMH I became aware of the fact that it was a very happy place. We had a number of new staff: I recall in particular a receptionist and Ephrem Abate who did the accounts. The Officers would come in regularly and the publications team were based there. It was an especially happy time of my life largely thanks to the quite outstanding support that I received from the officers so ably assisted by Susan Oakes.

With the benefit of hindsight, moving from Burlington House to DMH was a huge step forward for the LMS in terms of what it could undertake: we could do so much more with the increased administration, and the huge capacity of the building enabled us to both run and host meetings that we could never have had before. Perhaps the best example of this was the running of the International Review of Mathematics at DMH: there seemed to be a room for every need. The highlight, for me, was when one of the International Reviewers asked Lord May if they might speak with the Chief Mathematician at GCHQ. Lord May said he would arrange this. To our astonishment some 30 minutes later, when we were back in full session in the Council Room of DMH, Susan came in and announced that she had received a call from No 10 Downing Street to assure us that the Chief Mathematician Cliff Cox was on his way!

My summary view is that the move to DMH was both an historic and a very successful development for the LMS.

Everything you need to know about the REF

"Dear X, I am a PhD student/postdoc. I've heard of the REF, but I don't know much about it. What do I need to know?" — We invite two experienced academics to comment.

Cathy Hobbs *is Associate Dean for Research, Faculty of Environment and Technology, UWE Bristol, and Professor of Mathematics. She has extensive REF experience, including of overseeing Units of Assessment at Faculty level.*

The REF (Research Excellence Framework) is a mechanism for the research funding agencies of the UK Government to assess the quality of the research going on in UK universities. It is done by subject areas (called 'units of assessment'), of which Mathematical Sciences is one. It takes place every 6-7 years and is used to determine block research funding allocated to each university by the UK higher education funding bodies for the following 6-7 years. It is carried out by peer review of three aspects: (1) a sample of published outputs from each university, (2) brief descriptions of the impact our research has had on the economy and society (Impact Case Studies), and (3) a description of the research environment in each university subject area, including numbers of PhD students, funding won externally, equality and diversity actions taking place, and support for early career staff.

The REF is not an assessment of individuals so you won't need to submit a CV. It is very possible that papers you have authored/co-authored will be put forward within the sample your university will select and submit in December 2020. These can be papers you published before joining your current institution. Your university must submit a sample of outputs 2.5 times the number of full-time-equivalent staff deemed to be independent researchers in each unit (this will not include PhD students but may include postdoctoral researchers). Papers will be individually scored by the expert review panel during 2021, but only a profile of the scores for each unit at your institution will ever be public, not these individual scores. Getting a good overall score is important for your university as it will (1) determine much of its future direct funding (not including competitive funding coming via the EPSRC or Innovate UK), and (2) influence greatly its international reputation and hence its ability to attract research students and the best staff.

You may be asked to provide information about your publications since January 2014, from which your university will then choose a sample of between 0 and 5 to include within the submission.

You may find knowing the REF profile of your current university, or of another university you may be considering joining, a useful way of judging their relative research credentials. Within a subject area it is generally regarded as a reasonable proxy for research quality — not perfect, but a good indicator. But bear in mind that some less research-intensive universities or smaller departments can have pockets of real excellence that the REF score doesn't pick up. Trust your own judgement — metrics aren't everything.

Simon Blackburn *is Professor of Pure Mathematics at Royal Holloway, University of London, and a Mathematical Sciences sub-panel member in REF 2014 and 2021.*

Every UK institution is concerned about their research performance in the REF, because it brings funding and (importantly) it is a very visible measure of reputation. If your career aim is to get a permanent academic position, it is definitely worth knowing what your contribution to this process will be, and how it is assessed.

You will likely contribute your best research outputs to the REF. In Mathematics, this almost always means published papers (though what counts as an output is very broad, and certainly includes patents, for example). Fortunately for you, a good collection of

outputs for the REF is very similar to a collection of outputs that will impress a recruitment or promotion panel. First, quality over quantity: REF 2021 accepts at most five outputs per researcher, produced since late 2014. Second, well-written: when you write a paper, show your work at its best by taking the time to motivate your results and point out the interesting new tricks and techniques you have come up with. Don't write just for the 20-or-so experts in your sub-area, but for the thousands of interested mathematicians out there!

An aside about writing papers: always acknowledge your funder, and quote your grant number. It is nice to say thanks, and to demonstrate the great things that are coming from the money they are giving you (and so they fund you next time you ask!).

One thing you might not be aware of: there is an open access requirement for (most) REF outputs. The full details are quite complicated (the REF 2021 website has details), but to be sure your output is eligible you should make it freely available on your institution's research repository within three months of acceptance by a journal. An exception to this rule: the journal publisher can impose an embargo period of up to 12 months for Maths submissions. I would also make a habit of uploading your paper to arXiv: more people will read it.

Another way you might be asked to contribute is by helping to write an impact case study. A case study is a short account that starts with research you have been involved with, and ends with 'impact' (roughly speaking, non-research consequences of your research). Examples of impact might be: outreach activities; a company making or enhancing a product; an organisation changing its behaviour; the creation of innovative teaching methods. Non-examples of impact might be: lots of academic citations, or prizes, for your papers. The REF requires many fewer case studies than outputs, so most people will not be involved in preparing a case study. However, if you are, then it is very important: the impact needs to be well-documented and the links to your research convincing.

Now (I am sure you will not take offence) I am aware that I am replying to a question posed by an abstract ideal of a PhD student or postdoc: you are not a real person. But, reader, you are real. If you are also a PhD student or postdoc, I send you my best wishes for your future research career. I hope it will be as exciting and fulfilling as you hope, and as I know it can be.

Bridget Riley Retrospective Exhibition

Hayward Gallery, Southbank Centre, London, until 26 January 2020

Review by David Chillingworth

Installation view of Bridget Riley, *Movement in Squares*, 1961 at Hayward Gallery 2019. © Bridget Riley 2019, Photo Stephen White & Co

A student mathematical friend and I once enjoyed arranging pawns on a chessboard in a highly precise symmetrical alignment, and then very slightly displacing one of the pawns. A non-mathematical onlooker was quite baffled by our amusement. It touched on questions of the nature of symmetry and regularity, visual expectation, comfort and discomfort.

The work of Bridget Riley confronts these notions head-on. From her early work with black-and-white stripes, rectangles and zigzags, such as *Movement in Squares* (1961), to her recent calmer images with regularly spaced (but not quite regularly coloured) discs, such as her *Measure for Measure* series taking inspiration from Seurat, the artist subverts regularity and periodicity to extraordinary effect, sometimes by subtle perturbation, sometimes by brutal disclination.

For the three-dimensional piece *Continuum* (1963/2005), the viewer walks into a spiral and is embraced by a curved surface of collapsing angular stripes (Riley later considered this too literal and abandoned the idea, preferring to reach three-dimensional disorientation by two-dimensional means). The same hall devotes an entire wall to *Composition with Circles 4* (2004), a regular yet irregular pattern of large intersecting circles, reassuring and unnerving. How did the gallery install large wall paintings, already *in situ* elsewhere? The answer is that the artist repainted them, over a period of several weeks. This applied in particular to the bright, colourful and curvy *Rajasthan* (2012), which seems almost to hover in space in front of the wall. Indeed, many of the works can, technically, be reproduced, given that they are realised from meticulously annotated plans with precise details for construction, gradings of distance, alignment, colour shade and intensity. One hall displays a range of these blueprints showing the geometry of the constructions; there is much for mathematicians to appreciate here. How did the artist draw such accurate sine curves, I wondered?

Installation view of Bridget Riley, *Rajasthan*, 2012, at Hayward Gallery 2019. © Bridget Riley 2019, Photo Stephen White & Co

A work I particularly liked was a large canvas, *Static 4* (1966), of regularly spaced black dots on a white background: think of the Gaussian integers. Despite the pleasing and familiar periodicity, there was a sense of unease; closer scrutiny showed the rectangles to be less than square near the edges of the area while the dots were in fact nearly circular ellipses with slowly varying orientation. The artist is several steps ahead of you: what you see is not what you get.

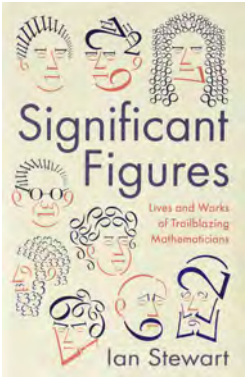
David Chillingworth

Dr David Chillingworth is a Visiting Research Fellow within Mathematical Sciences at the University of Southampton.

Significant Figures: Lives and Works of Trailblazing Mathematicians

by Ian Stewart, Profile Books, 2017, £10.99, ISBN: 978-1781254301

Review by Tony Mann



I was one of the many potential mathematicians whose youthful interest in the subject was developed by reading E.T. Bell's 1937 book *Men of Mathematics*, a collection of biographies of some of the great mathematicians of history. Although Bell's book has been criticised for confusing anecdote with history, a book which inspired mathematicians such as Julia Robinson, Andrew Wiles and John Nash was certainly important for twentieth-century mathematics. Ian Stewart's new book provides a 21st-century equivalent, and is likely to have a similarly positive effect in inspiring future generations of mathematicians. (And it has a rather nice title!)

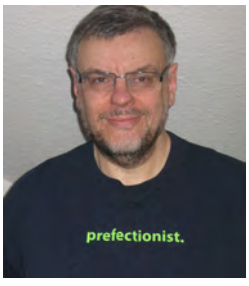
Naturally a point of interest, eighty years on, is who is "in" and who is "out". Only Archimedes, Fermat, Newton, Euler, Fourier, Gauss, Lobachevsky, Galois, Boole, Riemann, Cantor, Kovalevskaia, and Poincaré are common to both books. Such as Pascal, Descartes, Leibniz, Cauchy, Lagrange and several others don't make Stewart's cut, while Liu Hui, al-Khwarizmi, Madhava, Cardano, and Lovelace are selected by Stewart but not Bell. Stewart also naturally includes more recent mathematicians – Hilbert, Noether, Ramanujan, Gödel, Turing. Mandelbrot and Thurston. The reader making comparisons will reflect on how these differences may reflect changes in our view of mathematics over the last eighty years.

Stewart (as always) writes beautifully, and is more historically accurate than Bell, without ever sacrificing readability. His book is a lot shorter — 305 pages (in a larger font) against 591 in a recent edition of Bell — which I feel is no bad thing. His mathematician's insights are valuable, and the book's introduction and conclusion put his biographies into context. This is a book which anyone interested in mathematics will enjoy.

The cover of *Significant Figures* presents portraits by Steve Panton of several of the mathematicians discussed inside, which are remarkable (and appropriate) in being made up from digits — I am sure many others will get pleasure, as I did, from identifying them. But I have a criticism regarding some of the other illustrations. Each chapter is headed by a picture of the mathematician featured, but, unless I am missing something, no source is indicated for these. For a book which is likely to be read by many students, this is a terrible example to set – an undergraduate who failed to credit the images they used in this way would find themselves facing a serious charge. There may have been an error in production, but it is very unfortunate that these illustrations are not attributed.

That apart, I would want any mathematics undergraduate, or keen A-level student, to read this book. Inevitably, while *Significant Figures* happily features more diversity than Men of Mathematics, as a book about historical figures (none of whom was born more recently than 1946) its subjects do not represent the diversity of today's mathematicians, so I would like to recommend that readers also investigate Talithia Williams's *Power in Numbers: the Rebel Women of Mathematics*, a very different style of book, but one which provides a wider range of role models for prospective mathematicians.

Ian Stewart has written many valuable books, but if *Significant Figures* helps attract young people to mathematics, it could be one of his most important.



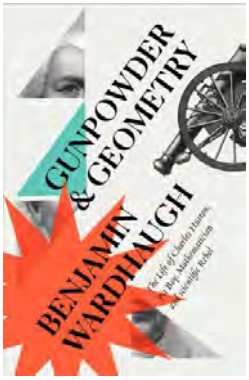
Tony Mann

Tony is Director of Greenwich Maths Centre at the University of Greenwich which works to promote the study of mathematics at all levels. His research interests include topics in the history of mathematics.

Gunpowder & Geometry: The Life of Charles Hutton, Pit Boy, Mathematician and Scientific Rebel

by Benjamin Wardhaugh, William Collins, 2019, £20, ISBN: 978-0008299958

Review by Chris Hollings



Charles Hutton's name is one with which historians of mathematics have long been at least vaguely acquainted, perhaps through his monumental *Mathematical and Philosophical Dictionary* [1], or through the many textbooks that he published during his lifetime. Further details of his life and work have been touched upon by those interested in late-eighteenth- and early-nineteenth-century mathematics, but this book — the first full biography of Hutton — brings this interesting figure to a much wider readership.

Hutton was born in Newcastle in 1737, and — as the title of Wardhaugh's book tells us — spent an early part of his life working down a Tyneside coal mine, whilst also attending a school run by a local clergyman, Jonathan Ivison. So impressive a student was Hutton, that when Ivison decided to stand down as schoolmaster in 1756, Hutton was his natural successor.

In the years that followed, Hutton continued to study more advanced mathematics, which enabled him to expand his teaching beyond the basic reading, writing, arithmetic and divinity with which he began. In this way, he built himself a reputation as an excellent teacher, and the number of his pupils grew. During this period, Hutton published his first textbook (on arithmetic, in 1764), and contributed dozens of solutions to the mathematical problems that were set in *The Ladies' Diary*, an annual almanac that he would go on to edit.

Such was Hutton's reputation as a teacher and an author by the early 1770s that when the position of Professor of Mathematics at the Royal Military Academy at Woolwich became available in 1773, his London contacts pressed him to apply. His application was successful, and he would hold the position until his retirement in 1807.

Wardhaugh's book details the many roles that Hutton took on at Woolwich: not only did he teach mathematics, and continue to write about it at various levels, but he also engaged in ballistic experiments (hence the 'gunpowder' of the title), and became the go-to mathematical consultant. Further recognition of Hutton's abilities came in 1774 with his election to Fellowship of the Royal Society, and the award four years later of its Copley Medal for a paper on ballistics. Also via the Royal Society, Hutton was heavily involved in calculations linked to the Schiehallion experiment to determine the density of the Earth.

Hutton's time as a Fellow of the Royal Society was not without its controversy, however. At the end of the 1770s, he was elected both to the Society's Council and also to the post of Foreign Secretary (despite his ambitions to the more prestigious role of Secretary), but his position became difficult following the election of the botanist Joseph Banks as the new President of the Society in 1778. The precise details of the subsequent rift between Banks and Hutton remain unclear; there certainly seems to have been a simple clash of personalities, with Banks taking against Hutton's outspoken manner. Snobbery on Banks' part towards Hutton's humble background, and the former's animosity regarding mathematics, probably also had a role to play. Whatever the cause, the effect was that Hutton and his (largely mathematical) supporters withdrew from the Royal Society, which then had little place for mathematics in the following decades.

Even without his position at the Royal Society, Hutton remained a prominent mathematical figure in Georgian Britain, with continued success in the fields in which he had made his name, that is, teaching and writing. During his later years, Hutton spent much time worrying about this legacy, and was particularly concerned that his role in the Schiehallion experiment should not be forgotten.

Upon his death in 1823, tributes were made by statesmen and scientific figures alike, but then as the nineteenth century progressed, Charles Hutton's renown gradually faded. Nevertheless, he is an intriguing figure, both in himself, and as a representative of his times, and so this biography is particularly welcome.

The picture that Wardhaugh paints of Hutton is a very human one that goes beyond mathematics. We learn about the twists and turns of Hutton's professional life, including some of the wrong moves that he made during his career. Nor are the details of Hutton's family neglected, insofar as these have been recorded. Indeed, one of the aspects of the book for which Wardhaugh is to be applauded is the extent to which he has been able to give a voice to the women in Hutton's life — this in spite of severely restricted historical sources relating to them. Wardhaugh highlights once again the part that women have often played in the mathematics of centuries past, not as research mathematicians, but in the equally valuable and usually overlooked role of editors and calculators.

Thanks to its compelling subject matter and engaging style, this is a difficult book to put down. Together with the accompanying volume of Hutton's correspondence that Wardhaugh [2] has also recently published, this book is a valuable resource for anyone interested in the state of mathematics in Georgian Britain. I strongly recommend it to both academic and general readers.

FURTHER READING

[1] C. Hutton, A Mathematical and Philosophical Dictionary, 2 vols., London, 1795.
[2] B. Wardhaugh, The Correspondence of Charles Hutton: Mathematical Networks in Georgian Britain, Oxford University Press, 2017.

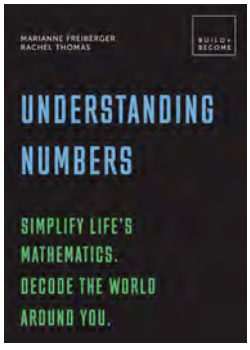
Chris Hollings

Chris is a Departmental Lecturer in the Oxford Mathematical Institute and a Senior Research Fellow of The Queen's College, Oxford. His research is in the history of nineteenth- and twentieth-century mathematics.

Understanding Numbers

by Marianne Freiberger and Rachel Thomas, White Lion Publishing, 2019, £9.99, US$15.00, ISBN: 978 1 78131 815 7

Review by Trevor Hawkes



I will try to answer three questions a book reviewer should ask: What's in the book? Who is it intended for? Should you buy it?

The authors, Marianne Freiberger and Rachel Thomas, are editors of the excellent PLUS magazine, a 'maths-awareness' website published by the *Cambridge Millennium Mathematics Project*. The following statement on the inside cover summarises their book's ambitious purpose: "In 20 dip-in lessons *Understanding Numbers* explains how and why mathematics fuels your world and arms you with the knowledge to make wiser choices in all areas of your life".

The lessons, as the authors call them, are not especially didactic. They are best viewed as short introductory essays to quicken interest. They are typically 2–4 pages long and are grouped into 5 areas.

- **Health:** The discussions of vaccination, screening, clinical trials include some simple mathematical models and statistics, and the final lesson is a 'health warning' on the misuse of statistics in the media.

- **Environment:** The Sydney Opera House sails and the London Olympic velodrome designs are shown to have geometrical solutions in the architecture lesson. There are lessons on traffic flows and modelling climate change, preceded by one on the difficulty of prediction in chaotic systems.
- **Society:** A lesson on averages is followed by one on fair voting systems – Arrow's theorem tells us there isn't one! The other lessons focus on the reliability of DNA evidence in the Courtroom and on how to manage your savings using knowledge of simple and compound interest.
- **Relationships:** The themes are the evolution of human kindness, choosing a partner, the statistics of sexual attitudes and behaviour, and the family tree.
- **Communication:** The first lesson on networks covers scalability and resilience as well as social networks; for instance why, on average, most people's 'friends' have more friends than they do. The other lessons are internet security (e.g. choosing passwords and RSA encryption), big data and artificial intelligence, and finally GPS location.

The condensation required in such a smörgåsbord of varied material runs the risk of cramming too much in. The authors have been disciplined in limiting their threads on each topic to a few salient, approachable ideas while ensuring the reader is given just enough information to understand them. The compression does occasionally come at a cost; I found myself slightly confused in the lesson on DNA evidence, and I was sorry that the lesson on GPS didn't mention the importance of special and general relativity for maintaining accurate clocks in the geostationary satellites.

The book should be largely accessible to an enthusiastic mid-teen working towards GCSE Mathematics. A few very simple equations are scattered sparsely throughout and there are suggestive, non-technical, diagrams on most pages, although they are not always well integrated with the text. The book is one in the London publisher's series called, somewhat vacuously, "Build+Become". I became allergic to the expansive and modish graphic design of the series, which does little to enhance the content.

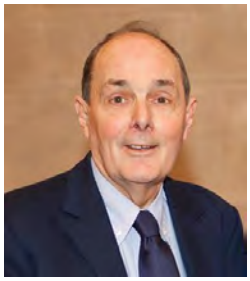
Should you buy it? As a reader of the LMS *Newsletter*, probably not for new mathematical insight, although it might incline you to look more closely at some of the topics via the further reading lists at the end of each themed section. It could, however, make a welcome gift for a younger family member who is less than inspired by mathematics as taught at school, or for one of those friends bold enough to ask you, as a professional mathematician, what you do with your time and why you do it.

Trevor Hawkes

Trevor, born near London Bridge, sees himself as a deculturated cockney who enjoys problem-solving and feels strongly about good mathematics teaching. His academic career included Cambridge, Uganda, Warwick (40 years), and Coventry University (8 years). He and his wife, Alyson Stibbard, each have Erdős number 3 and together they have four sons.

Obituaries of Members

Elmer Rees: 1941 – 2019



Elmer Rees, who was elected a member of the London Mathematical Society on 18 April 1974, died on 4 October 2019. He passed away peacefully after a long period of serious ill health. Over the years he served the Society in several capacities, notably as Vice President (1994–1996), and as a member of LMS Council. He was also the LMS–NZMS Forder Lecturer in 1995.

John Jones writes: Elmer was born in Llandybie in Carmarthenshire, West Wales, on 19 November 1941. He was a Welsh-speaking Welshman and this was very important to him. He was an undergraduate in Cambridge and a PhD student at the University of Warwick where his supervisor was David Epstein. In the first ever photograph of the Warwick Mathematics Department (in 1965) there are 12 people, Elmer is in it, so is David Epstein, and so too is David's supervisor, Christopher Zeeman. Three mathematical generations in a department of 12 people!

From Warwick, Elmer went to the University of Hull, the Institute of Advanced Studies in Princeton, and Swansea University, and then to St Catherine's College, Oxford in 1971. He left Oxford in 1979 for a professorship at the University of Edinburgh, where he stayed until 2005. From 2005-09 he was the Founding Director of the Heilbronn Institute for Mathematical Research, finally he was an Honorary Visiting Professor at the University of Bristol. In 2009 he was awarded a CBE in the Queen's Birthday Honours. One of Elmer's lasting contributions to the Mathematical community was his role in establishing the International Centre for Mathematical Sciences (ICMS) in Edinburgh in 1990, indeed in many ways ICMS owes its existence to Elmer.

Elmer was always tremendous fun to be with. He had an infectious sense of humour and a seemingly endless collection of anecdotes. Combined with an underlying seriousness of purpose, this made him a wonderful person to work with. His research interests were in geometry and topology and his publications cover the full range of this broad field. He really had the knack of picking good problems and crucial examples. For instance, his work on vector bundles over projective space, published in the 1970's, is now resurfacing in motivic homotopy theory. He had a characteristically concise style of writing. He seemed to be able to tell you exactly what you wanted to know with the minimum of fuss: his book 'Notes on Geometry' is a perfect example. Mathematical discussions with him were the same except that there was plenty of banter, jokes, and one-liners to keep everyone sharp.

Elmer was my doctoral supervisor in Oxford, indeed I was his first graduate student. I personally have many things to thank him for. He gave me a wonderful start to my career and he was never short of advice and opinion on all matters including mathematics, life, and Welsh rugby. I will always remember the 'rugby seminar' in Edinburgh. Shortly after Elmer went to Edinburgh I got an invitation to give a seminar on a Friday, which happened to be the day before the Wales – Scotland rugby match in Edinburgh. Elmer told me "of course I've got tickets for the match". This 'rugby seminar' continued as long as Elmer could get tickets. What a way to spend a weekend — mathematics and rugby in the company of Elmer. I will really miss him.

William J. Harvey: 1941 – 2019

Bill Harvey, who was elected to the London Mathematical Society on 18 November 1965, died on 22 October 2019, aged 77.

David Singerman writes: Bill attended the University of Birmingham and in 1962 began a PhD there with Murray Macbeath, then one of the UK experts in Riemann surfaces and Teichmüller theory. A fellow PhD student was Colin Maclachlan and both Bill and Colin wrote PhD theses on automorphism groups of Riemann surfaces. Bill made a fundamental study of the actions of cyclic groups and introduced Harvey signatures of Fuchsian groups. Bill and Colin would remain friends and collaborators until Colin died in 2012.

A world centre for research on these topics at the time was at Columbia University in New York where Lipman Bers was working, and this is where Bill went in 1966 after his PhD. New York was very important

to Bill for many reasons. Academically, he would become an active researcher in Teichmüller and moduli spaces, nourished by his participation in the Bers weekly seminar. Many of his colleagues would become lifelong friends and collaborators. Personally, he met his future wife Michele Linch, who was a student of Bers, and they remained together ever since.

In 1972, Bill returned to the UK where he took up a position at King's College, London. He enjoyed teaching and the company of his students, on whom he lavished a great deal of attention. He initiated weekly research seminars, realising the importance of seminars and conferences to the well-being of the research community.

At the 1978 Stony Brook conference, Bill gave a talk which introduced his curve complex. This is a simplicial complex associated to a surface S which encodes the combinatorics of simple closed curves on S. It turned out to be fundamental in studying the geometry of Teichmüller space. It was exploited afterwards by many mathematicians, and is now an essential tool for researchers in mapping class groups and hyperbolic 3-manifolds. Another important work used in subsequent research was his paper on branch loci in moduli space.

In 2007 Martin Bridson wrote: "Bill worked tirelessly over many years to maintain a geometry and topology seminar in London. Throughout that time he shared many insights with visiting researchers and always entertained them generously. His mathematical writings display the same generosity of spirit; he has written clearly and openly about his ideas rather than hoarding them until some arcane goal was achieved. The benefits of this openness are most clear in his highly prescient and influential papers introducing the curve complex."

Bill retired in 2008 but continued to engage in research, attending conferences, editing and reviewing for various journals and participating in the reading group he had begun with Shaun Bullett. Ever supportive of others' endeavours, Bill was commenting and collaborating upon mathematical work into 2019, until his illness made it impossible.

Throughout his career, most important were his students. In 2007 Bill wrote in the *Geometry of Riemann Surfaces*, "For my part, nothing in my career has given me more pleasure than to help my students find their feet in this corner of mathematics. It is a further source of satisfaction to see my former students, Christos Kourouniotis, Gabino Gonzalez and Paolo Teofilatto, take on the mantle of researcher and pass on the flame of mathematical excitement to their own students. Long may it continue."

In addition to his academic career, Bill was a very accomplished long-distance runner. He came 20th in the 1967 Boston Marathon with a time of 2h 29m 22s. In both New York and London he continued to run from home to his workplace, often to the astonishment of colleagues and students.

Bill will be sorely missed by his wife and children, Aileen and Daniel, his three grandsons, and the many mathematicians throughout the world with whom he interacted.

LMS Meeting

LMS South West & South Wales Meeting & Workshop

15 January 2020; 1:30 pm University of Bristol

Website: tinyurl.com/swsw20

The speakers at the LMS meeting are Martin Bridson FRS (Oxford), Corinna Ulcigrai (Bristol) and Yves Benoist (Paris). These lectures are aimed at a general mathematical audience and all are welcome to attend, including LMS non-members. The meeting forms part of the South West & South Wales Regional Workshop on *Interactions between Geometry, Dynamics and Group Theory* from 16–17 January 2020.

Funding is available for partial support to attend the meeting and workshop. Registration and funding at tinyurl.com/yyz2kuc6. The meeting will be followed by a reception. To reserve a place at the Society Dinner email lmsbristol2020@gmail.com.

Elmer Rees Memorial Meeting

Location: University of Bristol
Date: 10 February 2020
Website: tinyurl.com/vdeouv7

The Heilbronn Institute will hold a one-day conference to remember Professor Elmer Rees CBE and to celebrate his many contributions to mathematics on Monday 10 February 2020 in the School of Mathematics at the University of Bristol. This will include mathematical talks as well as personal reminiscences. Details are still to be confirmed, but further information will be published at the website above.

Mathematics Adapting to a Changing World

Location: Imperial College, London
Date: 26 March 2020
Website: tinyurl.com/yynzgpv3

This 6th IMA Mathematics Defence and Security Conference will bring together a wide variety of mathematical methods with defence and security applications. Aimed towards mathematicians, scientists and engineers from both industry and academia, in addition to government and military personnel who have an interest in how mathematics can be applied to defence and security problems.

LMS Meeting

LMS Invited Lecture Series 2020

30 March – 3 April 2020, Brunel University London

Website: boguslavsky.net/lms2020

The invited Lecturer will be Professor Yulia Mishura (Taras Shevchenko National University of Kyiv), who will talk on *Fractional Calculus and Fractional Stochastic Calculus, Including Rough-Paths, with Applications*. Accompanying Lecturers will be Elena Boguslavskaya (Brunel University London), Vassili Kolokoltsov (Warwick University), Nikolai Leonenko (Cardiff University), Joseph Lorinczi (Loughborough University), Hao Li (UCL) and Enrico Scalas (Sussex University). Funds are available for partial support to attend; email the organiser, Dr Elena Boguslavskaya (Elena.Boguslavskaya@brunel.ac.uk) with an estimate of expenses. Visit the website for further details and to register.

New Perspectives on SYZ Mirror Symmetry

Location: Imperial College London
Date: 30 March – 1 April 2020
Website: tinyurl.com/y45crs9x

The workshop will explore different approaches to the celebrated SYZ conjecture in mirror symmetry. The scientific program covers algebra-geometric, symplectic and non-archimedean perspectives in this context. There will also be specific talks aimed at general mathematical audience. More information can be found on the conference webpage. Supported by an LMS Conference grant.

UKACM 2020 Conference

Location: Loughborough University
Date: 1 – 3 April 2020
Website: tinyurl.com/wu6dxqb

This annual conference is a unique forum to present recent advances in all areas of computational mechanics. The conference promotes interdisciplinary collaboration, especially with computational mathematics, and includes a School on Advanced Topics in Computational Mechanics. Supported by an LMS Conference grant.

Counting Conjectures and Beyond

Location: INI, Cambridge
Date: 18 – 22 May 2020
Website: tinyurl.com/sqouozx

The announcement of the McKay conjecture in 1971 marked the origin of a different kind of counting conjectures of finite groups. The purpose of this INI meeting is to gather relevant mathematicians working on counting conjectures and related global–local problems in order to discuss the state of the art. Application deadline: 16 February 2020.

European Congress of Mathematics 2020

Location: Portorož, Slovenia
Date: 5 – 11 July 2020
Website: 8ecm.si/

At the 8th European Congress of Mathematics, Professors Stanislav Smirnov, Kathryn Hess, Bojan Mohar, and Robin Wilson will deliver public lectures and Sir Martin Hairer KBE FRS will deliver the Hirzebruch Lecture. Twelve prizes will be awarded at the Congress. Early bird registration and accommodation rates are available until 31 January 2020.

UK Easter Probability Meeting

Location: University of Manchester
Date: 30 March – 3 April 2020
Website: man.ac.uk/9ECEvJ

A meeting for researchers working in any area of probability and its applications. Mini-courses will be presented on *Dimers and imaginary geometry* (N. Berestycki), *Mathematical models from population genetics* (A. Etheridge), and *Stein's method and applications* (N. Ross). Research students and early career researchers will have opportunities to present a talk or poster, and some funding is available for their support. Supported by an LMS Conference grant.

Group Theory at BMC 2020

Location: University of Glasgow
Date: 7 – 8 April 2020
Website: tinyurl.com/u9p6k8k

The 'Groups' Breakout Sessions at BMC 2020 will run as a two-day event of the North British Geometric Group Theory (NBGGT) seminar network. The organisers aim to be inclusive in their programme and connect with all aspects of group theory currently represented in the UK. Funding is provided via LMS Scheme 1 and Scheme 3 grants.

Mathematical Physics: Algebraic Cycles, Strings and Amplitudes

Location: INI, Cambridge
Date: 1 – 5 June 2020
Website: tinyurl.com/uwff42k

This INI workshop will bring physicists with strong mathematical backgrounds with a mix of mathematicians working on transcendental aspects of string dualities and applications to amplitudes in physics. The aim is to consolidate recent progress on several fronts. Application deadline: 1 March 2020.

Modelling in Industrial Maintenance and Reliability

Location: Nottingham
Date: 14 – 16 July 2020
Website: tinyurl.com/y6n9rr9w

This is the premier maintenance and reliability modelling IMA conference in the UK. It is an international forum for disseminating information on the state-of-the-art research, theories and practices in maintenance and reliability modelling and offers a platform for connecting researchers and practitioners from around the world.

Society Meetings and Events

January 2020

15 South West & South Wales Regional Meeting, Bristol

March 2020

3-13 Apr Invited Lecture Series 2020, Brunel University

April 2020

8 Society Meeting at the Joint BMC–BAMC, Glasgow

19-25 LMS Research School, Graph Packing, Eastbourne

May 2020

11-15 LMS Research School, Methods for Random Matrix Theory & Applications, University of Reading

July 2020

20-24 LMS Research School, Point Configurations: Deformations and Rigidity, University College London

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

January 2020

8-10 British Postgraduate Model Theory Conference 2020, University of Leeds (484)

15 LMS South West & South Wales Regional Meeting, Bristol (486)

20-23 Statistical Aspects of Geodesic Flows in Nonpositive Curvature, University of Warwick (485)

February 2020

10 Elmer Rees Memorial Meeting, University of Bristol (486)

March 2020

16-20 Interactions between Group Theory, Number Theory, Combinatorics and Geometry, INI Cambridge (485)

23-27 Algebraic K-theory, Motivic Cohomology and Motivic Homotopy Theory, INI Cambridge (485)

26 Mathematics Adapting to a Changing World, Imperial College, London (486)

30-1 Apr New Perspectives on SYZ Mirror Symmetry, Imperial College London (486)

30-3 Apr UK Easter Probability Meeting , University of Manchester (486)

30-3 Apr LMS Invited Lecture 2020, Brunel University London (486)

30-3 Apr Arithmetic Geometry, Cycles, Hodge Theory, Regulators, Periods and Heights, INI Cambridge (485)

April 2020

1-3 UKACM 2020 Conference, Loughborough University (486)
6-9 Joint BMC/BAMC Meeting, University of Glasgow (485)
7-8 Group Theory at BMC 2020, Glasgow (486)
19-25 Graph Packing, LMS Research School, Eastbourne (486)

May 2020

1-3 UKACM 2020 Conference, Loughborough University (486)
6-9 Joint BMC/BAMC Meeting, University of Glasgow (485)
11-15 Methods for Random Matrix Theory & Applications, LMS Research School, University of Reading (486)
18-22 Counting Conjectures and Beyond, INI, Cambridge (486)

June 2020

1-5 Mathematical Physics: Algebraic Cycles, Strings and Amplitudes, INI, Cambridge (486)
8-10 Young Researchers in Mathematics 2020, University of Bristol
11-12 Energy Efficient Computing IMA Conference, Bristol
24-26 Numerical Linear Algebra And Optimization IMA Conference, Birmingham

July 2020

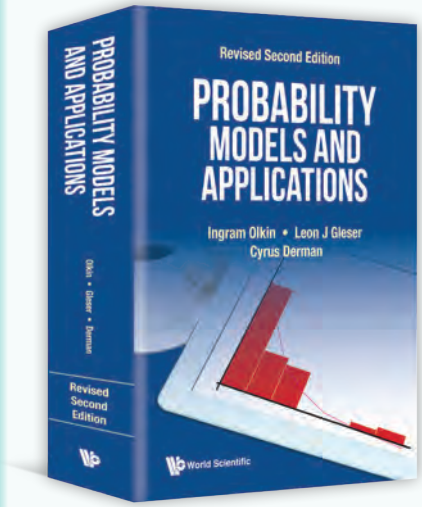
5-11 8th European Congress of Mathematics, Portorož, Slovenia (486)
12-19 14th International Congress on Mathematical Education Shanghai, China
14-16 IMA Modelling in Industrial Maintenance and Reliability Conference, Nottingham (486)
20-24 Point Configurations: Deformations and Rigidity, LMS Research School, University College London
27-7 Aug Integrable Probability Summer School, University of Oxford

August 2020

17-21 IWOTA 2020, Lancaster University (481)

September 2020

22-24 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge

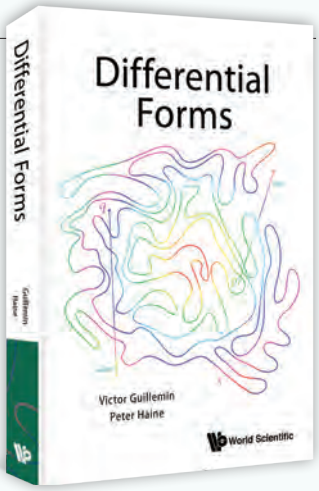


Probability Models and Applications
Revised 2nd Edition

by **Ingram Olkin** *(Stanford University, USA)*, **Leon J Gleser** *(University of Pittsburgh, USA)* & **Cyrus Derman** *(Columbia University, USA)*

Written by renowned experts in the field, this undergraduate textbook has as its unifying theme the role that probability models have had, and continue to have, in scientific and practical applications. It includes many examples, with actual data, of real-world use of probability models, while expositing the mathematical theory of probability at an introductory calculus-based level. Detailed descriptions of the properties and applications of probability models that have successfully modeled real phenomena are given, as well as an explanation of methods for testing goodness of fit of these models.

732pp | Sep 2019 | 978-981-3202-04-7(pbk) | £65



Differential Forms

by **Victor Guillemin** & **Peter Haine** *(Massachusetts Institute of Technology, USA)*

"Guillemin and Haine's goal is to construct a well-documented road map that extends undergraduate understanding of multivariable calculus into the theory of differential forms. Throughout, the authors emphasize connections between differential forms and topology while making connections to single and multivariable calculus via the change of variables formula, vector space duals, physics; classical mechanisms, div, curl, grad, Brouwer's fixed-point theorem, divergence theorem, and Stokes's theorem ... The exercises support, apply and justify the developing road map." CHOICE

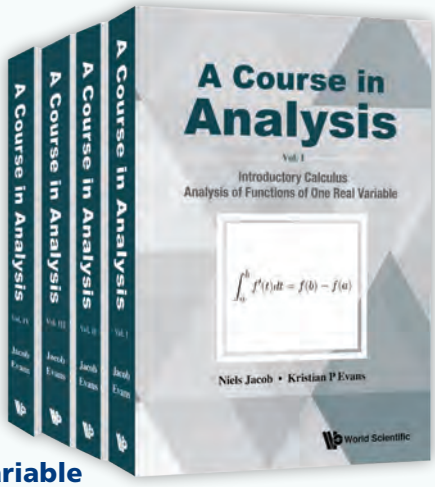
272pp | Mar 2019 | 978-981-121-377-9(pbk) | £40

A Course in Analysis

by **Niels Jacob** & **Kristian P Evans** *(Swansea University, UK)*

"The writing style is generally quite clear, and students should have little difficulty reading this book. The full seven-volume collection will no doubt be an indispensable reference for analysts and non-analysts alike." Mathematical Gazette

"The authors give many examples, illustrations and exercises to help students digest the theory and they employ use of clear and neat notation throughout. I really appreciate their selection of exercises, since many of the problems develop simple techniques to be used later in the book or make connections of analysis with other parts of mathematics. There are also solutions to all of the exercises in the back of the book. **A Course in Analysis** ***seems to be full of these little gems where the authors use the material or ask the readers to use the material to obtain results or examples that the reader will certainly see again in another context later in their studies of mathematics. I recommend these books."*** Mathematical Association of America Reviews



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